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Edited for the Council by J. LAURENCE PRITCHARD, Honorary Fellow

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NOTICES

Election of Members

The following members were elected at a Council Meeting held on March 20th :—

Foreign Member.—F. W. Follett.

Student.—John M. Radcliffe.

Silver Medal

The Council have awarded the Society's Silver Medal for 1922 to Lieutenant-Colonel E. W. Stedman, O.B.E., for his paper on "Some Technical Aspects of Aviation in Canada" published on page 349, Vol. XXVI., of the Journal.

Associate Fellowship Examination

The next examination for Associate Fellowship has been postponed from April to September next. The exact date will be announced later.

R38 Memorial Prize

The following members have been appointed to adjudicate on the papers submitted for the first award of the R.38 Memorial Prize :—

Professor L. Bairstow, C.B.E., F.R.S.
Wing Commander T. R. Cave-Browne-Cave, C.B.E., R.A.F.
Professor A. J. Sutton Pippard, D.Sc.
Major R. V. Southwell.

Students' Section

It is proposed to hold another Smoking Concert at the Engineers' Club, Coventry Street, W.1, at 6.45 p.m. on Friday, April 27th. The students hope that members of other grades will join them on this occasion. Tickets (price 1s. 6d.) may be obtained from the Secretary.

Students are invited to send in offers of papers to be read at Students' Meetings during next Session (commencing in October) to the Honorary Secretary, Students' Section.

Students' Visits

Thursday, April 19th.—Marconi Works, Chelmsford. Meet at Main Line Booking Office, Liverpool Street Station, at 1.55 p.m., for cheap travelling facilities.

Wednesday, April 25th.—Napier Engine Company, Acton. Meet at the Company's Works at 2.50 p.m.

Saturday, May 5th.—Vickers Ltd., Weybridge. Meet at Main Line Booking Office, Waterloo Station, at 9 a.m.

Those desirous of attending these visits should send in their names to the Honorary Secretary, Students' Section, at least four days beforehand in each case.

Forthcoming Arrangements

Tuesday, April 17th.—Council Meeting.

Thursday, April 19th.—Students' Visit to Marconi Works, Chelmsford.

Wednesday, April 25th.—Students' Visit to Napier Engine Co., Acton.

Saturday, May 5th.—Students' Visit to Vickers Ltd., Weybridge.

Thursday, May 31st.—5.30 p.m., Royal Society of Arts. *Wilbur Wright Lecture.* Dr. J. S. Ames, "The Relation between Aeronautical Research and Aircraft Design."

Correction

In R. A. Frazer's Juvenile Lecture in the March number, Figs. 3, 4 and 8 were unfortunately printed upside down.

W. LOCKWOOD MARSH, *Secretary.*

PROCEEDINGS

SEVENTH MEETING, 58TH SESSION

An Ordinary General Meeting of the Society was held at the Royal Society of Arts, John Street, Adelphi, London, on Thursday, January 18th, 1923, the Chairman, Professor Bairstow, in the chair.

The CHAIRMAN, after introducing Major J. D. Rennie, who was to read a paper entitled "Some Notes on the Design, Construction and Operation of Flying Boats," said the Paper was a very full one. In this connection he pointed out that the Council of the Society welcomed full papers of this description which were suitable for publication in the Journal of the Society, and that it was not necessary for authors, in preparing their papers, to make them so short that they could be delivered within the limits of a lecture. The Society welcomed Major Rennie, who had a unique experience of the technical side of the development of the British flying boat. He had been associated with the late Colonel J. C. Porte, C.M.G., at Felixstowe, who was chiefly responsible for the early development of the flying boat in the British Air Service. He (the Chairman) also welcomed the Paper, because, to some extent, it challenged outside designers as against Air Ministry designers, and in issuing the challenge—quite a moderate one—the Author had taken care to provide anyone who wished to reply with the necessary material. That, again, was good for the development of the subject. It was not usually a sign of good health that one should take peacefully and quietly all that one was told.

Major RENNIE then delivered his Paper, which he illustrated with a number of lantern slides.

SOME NOTES ON THE DESIGN, CONSTRUCTION AND OPERATION OF FLYING BOATS

Introductory

Since the Armistice, the development of the flying boat in this country has been practically at a standstill; this for two main reasons. Firstly, the unfortunate death of the late Col J. C. Porte, C.M.G., who was directly responsible for the development of the flying boat in the British Air Service, robbed the aeronautical world of one of its greatest pioneers in this branch of aeronautical engineering; secondly, as about 99 per cent. of the flying boats used on active service during the late war were designed, and the first of each type constructed and tested, at the Seaplane Experimental Station, Felixstowe, where the late Col. Porte was Commanding Officer, and the lecturer was for some time Chief Technical Officer, private firms had not the opportunity to gain the necessary practical experience obtained from extensive flying operations carried out under all sorts of conditions and weather.

The result of this lack of experience, which could only be acquired by private firms at a prohibitive cost, has been that flying boats produced by them during and since the war, with possibly one exception, have proved unsuccessful, and in several cases complete failures.

Such being the case, the chief object in writing this paper is to attempt to put on record the main results of the experience in flying boat design and construction obtained at Felixstowe and elsewhere, and to compare the merits and demerits of the other known types of hull construction with that developed there.

At the same time, in order to make the paper more or less complete, it is proposed to give a brief discussion of the various problems in design, apart from the hull, which have arisen from time to time and present difficulties unique to the flying boat.

Originally it was intended to treat in all its aspects the problems connected with the design, construction and operations of flying boats, but at the request of the Society such matters as model testing, operations, and others likely to be dealt with in the forthcoming lectures of Baker and Wing-Commander Cave-Browne-Cave have been dealt with less completely.

Some General Considerations

It is probably true to say the flying boat is the least known of all types of aircraft, very likely due to the fact that during the war their operations, mainly submarine patrols and ship escorting, were conducted from a few seaplane stations on the coast, and the Orkney and Scilly Islands. Little limelight was given to their achievements. In this connection, it may be mentioned, as a matter of interest, that flying boats operating from Felixstowe were responsible for the sinking of thirteen German submarines and the destruction of one Zeppelin. Also, during a period of eighteen months' operations from two seaplane stations, 289,000 sea miles were flown without the loss of a single boat, apart from enemy action, providing sufficient testimony to the sea-keeping qualities and airworthiness of these boats.

Again, commercial services so far have been confined to routes over the land or at most covering short stretches of sea, such as the English Channel, where no advantage would be gained by the use of flying boats. When, however, these services become firmly established and the consequent extensions entail long flights over the sea, the use of flying boats will become imperative.

It would seem as if such operations will be confined mostly to countries, as our Colonies, where there is an extensive coast line, or broad rivers extending far inland, or vast tracts of virgin forests with inland water, as in Canada, and where, under these circumstances, aerodromes are impracticable.

With the possible exception of to Norway and Sweden, the prospect of establishing long sea routes from this country are not hopeful at present. In the not too far distant future regular services to America will probably be available for those to whom the saving in time will warrant extra expense over the steamship. Such route would most likely be via the Azores Islands in the Atlantic.

Over long sea routes, say, 100 miles and upwards, the flying boat possesses overwhelming advantages over its only rival, the steamship, contrasted with the aeroplane competing with the train. In the former case, taking the maximum speed of a ship as 20 knots and the cruising speed of a flying boat as 80 knots, the voyage would be accomplished in a quarter of the time; in the latter case, the average speed of the train might be about 40 miles per hour and cruising speed of aeroplane 95 miles per hour, only slightly over half the time.

With regard to the effect of weather on operations, the conditions are more favourable to the flying boat. Where the operations are carried out from sheltered harbours the flying boat can take off in weather in which it would be dangerous for the large aeroplane to make the attempt. Night flying is also safer, as in case of emergency a safe landing can always be made, even with a moderately rough sea running, when the trouble may be located, and if possible repaired; otherwise the boat may be taxied so long as one engine is available to the nearest seaplane station, sheltered water, or steamship route.

In the event of fog, should it be necessary to land, this may be achieved reasonably safe (provided hull bottom is properly shaped) by means of a simple method of automatic control, which was successfully used early in 1915, and has

since been considerably improved. A point worth mentioning which is not generally realised is that new types of flying boats, up to any size likely to be built in the future, may be tested with safety before taking off the water to any extent. Choosing suitable weather and sea conditions, it is possible to plane at speeds below and well above the lowest flying speed of the boat, and then gradually to take off and fly over as much of the speed range as desirable while only a few feet off the water, thus enabling controls, trim, etc., to be sufficiently tested before actually gaining a height, such if trouble arose a dangerous situation might arise. This, coupled with the fact that should one engine cut out suddenly when taking off the consequences are not likely to be serious, the converse being the case with a multi-engined aeroplane, accounts for the total absence of any serious accident during the testing of experimental flying boats at Felixstowe.

For the above two reasons alone it would seem logical that the most suitable type of aircraft to develop in large sizes is the flying boat.

It has been remarked frequently by critics of the flying boat that in performance and general design it is inferior to that of the modern aeroplane.

Obviously, with regard to performance, this must be true of the single-engine flying boat, owing to the interplane engine mounting—an undesirable feature for which there would seem to be no practical alternative at present. Also the aerodynamical and detail design and general lay-out unfortunately is much inferior to that represented in the best modern aeroplane practice for no apparent reason other than lack of experience and ability on the part of the designers. The above, however, cannot be said of the multi-engined flying boat, as a few simple considerations may show. For the purpose of aerodynamic comparison, the trials of the F.5 flying boat and the Vickers Vimy commercial aeroplane carried out at the Air Ministry test stations at Grain and Martlesham respectively have been analysed. Both machines are fitted with two Rolls Royce Eagle VIII. engines, and are of approximately the same weight and class. Taking as a basis of comparison the lbs./h.p. carried at a speed of 95 m.p.h. at 2,000ft. with a loading of 9 lbs./sq. ft., the figures are 21.2 for the F.5 and 17.4 for the Vimy. The former had bare engines and the latter faired nacelles. The greatest uncertainty in the above analysis was the engine-power curve, as tests were not carried out at the time of the trials. The curve used was an average one from a large number of A.I.D. tests.

That it is quite reasonable to expect a result of this order may be seen from other considerations :—

TABLE I.

BODY RESISTANCE.

	Drag at 100ft./sec.
	Max. cross sectional area.
Streamline $4\frac{1}{2}$ fineness ratio	0.66
P.5 flying boat hull	1.38
F.5 flying boat hull	1.53
Bristol Pullman fuselage	1.90

In Table I. are compared the air resistance of the F.5 and P.5 flying boat hulls and the Bristol Pullman fuselage. The resistance of a good streamline form is also given as a matter of interest. The figures given are for model only, without wing interference; but assuming the effect is similar, it shows that a hull can be designed to have as good, if not a better, low resistance form in spite of the limitations imposed on the shape by hydrodynamical requirements than the corresponding aeroplane fuselage.

TABLE II.

PERCENTAGE WEIGHTS.

Aeroplane.	Gross Weight. lbs.	Lbs./sq. ft.	Lbs./H.P.	Wing Structure.	Tail Unit.	Fuselage and Tail Skid.	Chassis, or Wing Tip Floats.	Plant.	Supplies.	Structure.	Useful Load.
H.P. 0/400	13,200	8.00	18.4	16.4	1.5	11.4	5.5	22.3	2.5	34.8	40.4
H.P. 4.8 B.	11,553	7.92	16.32	17.8	1.54	12.3	4.36	21.64	1.37	36.72	40.2
H.P.V. 1500	30,000	10.00	20.7	17.45	1.71	5.7	4.50	19.11	2.01	30.06	48.9
Bristol Tramp	18,795	8.25	19.6	21.09	1.43	7.38	4.04	30.17*	2.16	35.82	31.8
Vickers Vimy	11,120	8.40	15.45	17.55	2.1	13.0	3.45	29.4	1.9	36.1	30.8
										Average	38.5
Flying Boat.											
P.S.B.	32,000	10.2	18.5	16.5	1.7	12.8	0.75	19.0	4†	33	44
F. 5	13,000	9.1	18.6	16.0	2.2	15.7	0.92	20.0	3	36	41
P. 5	12,055	9.4	17.02	15.4	3.1	15.5	1.22	22.56	2.41	35.8	39.3
Short Cromarty ...	19,633	8.7	18.7	19.01	2.21	14.7	0.80	22.93	2.0	37.32	37.8
										Average	40.5

NOTE.—Wing structure includes interplane engine mounting.
Fuselage does not include controls or instruments.
Hull includes wing roots, seats and bulkheads, but not controls or instruments.
Useful load includes fuel and oil.

* Includes engine shafting, couplings and gearing.

† Includes tankage for 1,500 gals. of petrol and 100 gals. of oil.

Further, at top speed the air resistance of a fuselage and tail skid is of the order of 12 per cent. and the undercarriage from 10 per cent. to 15 per cent., making a total of from 22 per cent. to 27 per cent. of the total resistance, whereas the resistance of hull, wing tip floats and wing root struts does not exceed 10 per cent. of the total.

Table II. gives some per cent. weight data of a number of aeroplanes and flying boats of approximately the same performance and class, which, with the notes attached, needs no further explanation. The main point to note is that the average useful load of the flying boat is 40.5 per cent. against 38.5 per cent. for the aeroplane.

TABLE III.

Aeroplane.	Weight lbs.	A Area Main Planes.	c Chord M.P. ft.	Area Tail Plane. sq. ft.	l. ft.	Tail volume ratio. $\frac{a_t \times l}{A \times c}$	Type of Tail Plane.
Victoria ...	17,700	2166	13′ 0″	214	31.5	0.239	Biplane.
Awang ...	17,015	2301	11′ 9″	229	41	0.339	,,
Braemar ...	16,500	1905	8′ 6″	180	36	0.395	,,
H. Page ...	13,200	1645	10′ 0″	184	44.9	0.5	,,
Vickers Vimy..	12,500	1329	10′ 6″	180	31.0	0.4	,,
Bolton ...	9,100	928	7′ 9″	100	27	0.376	Monoplane.
De H. 10 ...	8,500	851	7′ 0″	109	21.5	0.394	,,
Flying Boat.							
P.S.B. ...	32,000	3108	10′ 0″	378	33.75	0.41	Biplane.
N.4	32,000	2675	12′ 6″	368	37.7	0.41	,,
F.5	13,000	1432	8′ 0″	186	24.5	0.39	Monoplane.
P.5	12,055	1273	9′ 0″	196	25.0	0.42	Monoplane R.A.E. 15 inverted.
Valentia ...	20,650	2025	10′ 9″	296	29.5	0.4	Biplane.

Table III. gives some particulars of tail volume ratio and dimension l, the distance between the C.G. of the whole machine and C.P. of tailplane.

It will be noticed that the dimension l is smaller in the flying boat than in the aeroplane, while the chord of main planes in the former case is generally smaller, the tail volume ratio being of the same order in both types.

This dimension might be increased, but only at the expense of a considerable addition in hull weight, and as experience shows it does not seriously affect the stability or performance, the increase is not warranted. As will be shown later, l is not fixed from purely aerodynamical considerations.

Making due allowance for a certain amount of inaccuracy in the above data and tables, it is fair to say that a multi-engine flying boat can be designed to give as good a performance as the corresponding aeroplane, if not even better.

There are three types of flying boats in common use :—

(1) The more orthodox, of which the F.5 and P.S.B. are typical. It consists of a hull to which the wing structure and tail unit are attached direct. Interplane engine mounting and wing-tip floats.

(2) In this type the hull is shorter than in type (1). It is cut off at the rear step, and tail unit carried on outriggers. The American N.C.4 and R.A.E. C.E.1 are examples.

From the aerodynamic point of view, this type offers the best solution for a single-engine flying boat, as it is possible to place the fin and rudder symmetrically with respect to the rotational component of the propeller slipstream, thus

eliminating the necessity to carry rudder. Also, as the tail plane is higher up than in type (1), it is clear of wave formation set up when taking off.

(3) To avoid the necessity of using wing-tip floats, especially in the case where a cantilever monoplane wing placed relatively to the hull about where the top plane of normal biplane arrangement would be, is employed, the fin has been extended outwards symmetrically with respect to the C.G. of the boat in order to obtain a positive metacentric height. The extension may be either parallel to the water line or form a small cantilever wing set at a large dihedral angle, thus providing an increasing righting couple with angle of hull. It also contributes to the aerodynamic lift in flight.

It is with type (1) that this lecture is concerned, and in particular multi-engined boats having a displacement not less than 4,500 lbs.

Below this displacement the float seaplane would appear to have the advantage except under most exceptional circumstances, as when operations are carried out from sheltered inland waters. Unless the hull is much larger than other requirements would indicate, the pilot and passengers are liable to get a good wetting when taking off in a short sea causing great discomfort during the flight, especially in cold weather. On the other hand, if totally enclosed, there is a serious element of danger, owing to the flinging up of spray and solid water on the windows, which renders clear vision most difficult at a time when it is all-important.

Aerodynamic Structure

The usual biplane wing arrangement of a flying boat differs in at least two respects from that of the large aeroplane. The span of the top plane is considerably greater than that of the lower. This was not intended primarily to obtain greater aerodynamic efficiency owing to the absence of biplane interference on the extensions, but because it is advisable to have as small a wing area as can be conveniently arranged in close proximity to the water, thus minimising risks of danger in a rough sea. For the same reason ailerons should not be fitted to the lower plane. To make up for this, and to ensure adequate lateral control, the ailerons should be placed as far out board as possible.

Also, paying due regard to increase of dimensions and consequently weight of the wing-tip floats, the closer they are to the hull the less shock is transmitted to the wing structure when the boat is rolling, taking off or landing, one wing down. The result of these considerations led to the adoption of the wing arrangement as described.

In the case of the "Fury," which was a triplane, ailerons were fitted on the top and middle planes only. The latter were of equal span, and greater than that of the lower plane. It has been said frequently of the "F" boats that the use of stabilisers on the top plane is a very inefficient aid to lateral stability. They were never really intended as such, as it was well known an increase of dihedral would be much more effective. The top plane extensions had to be braced to withstand down loads which are applied as an air load at high speed flight as its own weight or as inertia loads caused by a bad landing. The rectangular cross-braced pylon was considered to be a more sound job than the triangular type, and was thus adopted. It was completely faired in, thus giving less air drag than would be given by the sum of the component parts.

While extensions have the disadvantage of requiring a large number of bracing wires, on the other hand, if the planes were of equal span, another set of interplane struts and wires would probably be required if normal wing sections were used.

In order to keep the propellers reasonably clear of spray or solid water, the distance of the line of propeller thrust from the C.G. is much greater than that of the aeroplane, in which by mounting the engines on top of lower planes this line may be made to pass nearly through the C.G. While disadvantageous from some points of view, it is not without its compensations. It is helpful to stability, and especially to absence of change of trim, engine off and on. By arranging the C.G. well forward, which means good longitudinal stability, other things being equal, there will be a down load on the tail over the speed range, which by suitable aerodynamic design will be further increased by the propeller slipstream, thus balancing the propeller thrust moment.

If accurate data were available as to the angle, the increase of velocity and the boundary of the downwash at the tail plane, a simple calculation would fix the range of adjustment and the position and area of the tailplane in the propeller downwash to obtain the necessary balance at at least one speed, say, cruising speed, and a fair compromise over the remaining speed range. Unfortunately this data is not known with sufficient accuracy.

Various formulæ have been devised, the best known being that given in R. & M. 326. Wind tunnel investigations have shown this formula may be an error to the extent of 2 degrees, under-estimating the deflection. Further, the same test showed that in the case of a tractor propeller the angle of pitch relative to the main plane chord had little effect on the angle of downwash on the tail plane. Mr. R. McKinnon Wood has suggested that the increase in the angle of downwash is due mostly to the increased speed of the air over the portion of the wings in the slipstream, thus increasing the lift, of which the downwash is a function, a view which receives considerable support from the Prandtl theory of circulation.

Again, the formulæ derived so far neglect the effect of the rotational motion of the air in the slipstream. For example, in the "F" boats the change of incidence of the tail plane to trim at the same speed, when for two engines turning in opposite directions were substituted two turning in the same direction, was of the order of $\frac{3}{4}$deg., which gives some idea of the error involved in this assumption.

In the "F" boats, which had twin tractor propellers, the tail plane of which, about three-quarters, was in the slipstream, was checked by the above formula and seemed fairly satisfactory, but on trial was decidedly tail heavy engine off, as were all the "F" boats. This may be partly accounted for by the fact that the C.G. when carrying full military load was further back than was originally intended.

With pusher propellers the conditions are obviously simpler, and the results of calculations likely to be more in agreement with the observed facts. On the other hand, pusher propellers on a flying boat are very liable to damage from parts which may either break off or work loose from nuts, and even spanners, etc., which have been left through carelessness in such a position that they fall into the propeller in flight. This has happened frequently in the lecturer's experience, no matter what precautionary measures are taken.

The most satisfactory, and really the only way at present, apart from full scale experimenting, which is both costly and wasteful of time, to ensure the correct disposition of the tail plane surface to fulfil the above requirements is to test a complete model in the wind tunnel, reproducing the actual slipstream by a propeller.

In making out a case for the tail plane supported on outriggers, it has been said that there is great difficulty in fixing the tail unit rigidly on a flying boat hull. Experience has not shown this to be the case. There is no difficulty in mounting either a monoplane or biplane tail of any aspect ratio and in any vertical position relative to the propeller axis likely to be required. With the

latter type it will usually be found necessary to have the lower plane of smaller aspect ratio than the upper one in order to avoid damage from the side waves which close in at vicinity of the tail when "taking off."

An adjustable tail plane may, however, present some difficulty, as in case of a biplane tail plane, of which the fins and rudders form an integral part; the effort required to operate it under its own weight, apart from air load, may be beyond the capability of the pilot, unless some auxiliary motive power is used, such as oil pump and ram. Anticipating this trouble on the "Fury," the elevator on the lower plane was operated separately by a long lever on a quadrant centred on the elevator control shaft. This was used to obtain trim and the upper elevator for extra control in the usual way. The result was quite satisfactory, but owing to hull interference the efficiency of the lower plane was relatively low.

With regard to fin and rudders, owing to the long forebody, these are of relatively large area. The rudder area on the "F" boats was barely sufficient for control with one engine cut out completely. The "Fury," however, was very satisfactory in this respect; the arrangement there used, namely, three propellers abreast, and three fins and rudders in their respective slipstreams, would seem to solve the problem of one engine cutting out. Little data appears to be available on the subject; calculations are liable to be greatly in error as the yaw taken up by the machine after one engine has cut out is difficult of determination with any degree of accuracy. A figure of some interest in this connection and not generally available is the air drag of a stationary propeller. Tests of a fair variety of different propellers gave from 12 to 13 lbs. per sq. ft. of projected area at 100ft./sec.

It is a matter of some importance to be able to estimate the aerodynamic balance and maximum moment which can be exerted by the elevators at or near the taking-off speed. Wind tunnel tests on a complete model show there is a slight increase in kl of the order of 10 per cent. over the usual range of angle of incidence, and a considerable effect on pitching moment, if the model is tested under conditions corresponding to the aeroplane being at ground level. The change in moment is obviously the result of the flattening out of the downwash at the tail plane due to the interference of the ground.

This produces a larger tail lift or nose-diving moment, becoming greatest at large angle of incidence. With a flying boat the conditions are more complicated, owing to the wave formations at the tail. With slipstream effect it would seem impossible to predict the angle of downwash at the last plane, and recourse must again be made to wind tunnel tests, reproducing the actual working conditions. Fortunately, flying tests have shown that aerodynamic balance can be obtained, as at least two types of boats have taken off into the air without the use of elevator control.

From the above brief discussion it will be observed that there are many problems of first importance in the aerodynamical design of a flying boat which, for a satisfactory solution, depend upon an accurate knowledge of the resultant propeller downwash at the tail plane. As the many complex factors entering into the complete equation are probably beyond mathematical analysis, and the various more or less empirical formulæ unreliable and likely to lead to disappointing results, the necessity for developing wind tunnel tests on complete models, propellers running at the appropriate advance per revolution, cannot be over-estimated.

As with the aeroplane, controllability at low speeds is naturally of great importance, but probably relatively less, as generally the choice and extent of a landing ground is not so restricted. Also, once on the water the boat pulls up quickly, owing to the large hull water resistance. Thus it is possible, and is

the usual practice, to glide at a comparatively high speed until close to the water before flattening out and alighting.

In connection with the operation of control surfaces on large flying boats, it may be of interest to state that on the Felixstowe "Fury" the rudder, ailerons, but not the elevators, were balanced; servo-motors were also fitted to all the control surfaces as it was thought adequate control under certain flying conditions might not be within the capability of the pilot, there being no previous experience with a boat of this size. However, she proved to be extraordinarily light on all the controls and superior to the much smaller "F" boats. The servo-motors were therefore removed, as Col. Porte decided that any advantage to be gained by their use did not warrant the additional weight and complication.

Hull Lines and Dimensions

The setting out of the lines and fixing upon the main dimensions of a flying boat hull to meet the requirements of a given h.p. and loading specification is a problem by no means easy, owing to the many varied and often conflicting conditions which have to be fulfilled. A large amount of data from tank tests on models is available, especially with regard to the effect of modification in form and dimensions; but it should be remembered that such tests are carried out under smooth water conditions, under which conditions it would be expected that full scale and model would be in agreement; such conditions in actual practice are the exception rather than the rule. Tank tests, however, afford the only reliable means of obtaining data with regard to resistance, hydroplaning efficiency, etc., and the effects of minor alteration in form of which the designer must have full information. The development of the technique of tank testing is, in this country, due almost entirely to the work of G. S. Baker of the N.P.L. Probably the main point in which the experience gained on full scale work is at variance with the results of tank tests is in the dimensions of a hull for a given displacement, about which there has been much controversy, and finality has not yet been reached.

The most successful design will result from the best compromise between the various water and aerodynamic requirements, and will depend primarily upon sound judgment based upon the results of considerable full scale and tank experiments.

The modifications carried out on and the new types of hulls evolved at Felixstowe were arrived at from full scale experimenting, and with the exception of the "Fury" hull, tank tests on the corresponding models were not available at the time. In so far as the conditions may be compared, with the possible exception of the relation of dimensions to displacement, the deductions from full scale are in agreement generally with those from the model.

As facilities were not available to measure the full scale resistances, the criterion adopted was the lbs./h.p. at which a boat could be taken off. This was justifiable, because in many cases the same or a similar type of superstructure was attached to different hulls. Admittedly this was a rough and ready method, although quite rational under certain assumptions. With experience it was possible in this way to predict with fair accuracy the loading capacity of any proposed type of hull design.

As the hydrodynamic forces acting on a hull during the period of taking off are even more complex than the corresponding aerodynamic forces acting on a body, recourse must again be made to model testing. The principles involved in the transition from model to full scale will now be discussed briefly for the sake of interest and completeness.

The resistance to the passage of a hull through the water is made up of two parts, frictional and wave making. The water forces may, therefore be assumed

to depend upon the density ρ, the speed v, a dimension l, usually the length between perpendiculars, the kinematic viscosity γ, and acceleration due to gravity g. Accepting such, it is easily proved from the theory of dimensions that the resistance R must take the functional form

$$R = \rho v^2 l^2 \ (vl/\gamma \ gl/v^2) \quad . \quad . \quad . \quad . \quad (1)$$

in which (vl/γ) (gl/v^2) are the friction and wave making terms respectively. Many years of ship design experience and experiment have shown that these terms may be treated separately, and further, in the case of hydroplanes, Baker has shown that skin friction may be neglected.

We may therefore write

$$R = \rho v^2 l^2 \ (gl/v^2) \quad . \quad . \quad . \quad . \quad . \quad (2)$$

which is the basis of Froude's Law of Corresponding Speeds, at which speeds the wave formation of full-scale and model are similar. Denoting full-scale by capital letters, we have from (2) above

$$R/V^2L^2 = \gamma/v^2l^2 \quad . \quad . \quad . \quad . \quad . \quad (3)$$

provided $V/v = \sqrt{(L/l)}$ and the under-water parts geometrically similar, that is, attitude of full-scale and model identical, and displacements in the ratio of the cube of the linear dimensions. If Δ is the displacement at rest, and P at any other speed V, then assuming the whole load air-borne at the taking-off speed, and air load at any other speed proportional to the square of the velocity, as the attitude is nearly constant, P is readily obtained, since $P = \Delta$ — air load.

In the same way it may be shown if M is the full-scale moment

$$M/m = (L/l)^4$$

It may happen, and frequently does, that, in a proposed design of hull, the under-water lines are similar to a hull of which only the full-scale resistances and moments are known. To this case the above applies also, further noting the taking-off speeds will be in the ratio $(V/v)^{\frac{1}{2}}$.

As an illustration, Fig. 1 shows typical water resistance and attitude curves under smooth water, no head wind, and natural trim conditions for a twin-engine flying boat hull of 12,500lbs. displacement. The air drag has been calculated, allowing for slipstream effects and added to the water resistance curve giving the curve of total resistance. In Fig. 2 these results are exhibited in a more interesting form, to which has been added the propeller thrust horse-power available. Under these conditions, which are the most adverse from the h.p. point of view, obviously the criterion that the boat will take off is that the curve B lies wholly below the h.p. available curve, also as the thrust available = water + air resistance + mass × acceleration it will be seen that the area between these two curves up to the ordinate at the getting-off speed is proportional to the length or time to unstick, provided the hull is running at its natural trim.

In order to help toward a clearer understanding of the discussion which follows, it is now proposed to give a brief description of a process of taking off, and the action of the hull under normal sea and weather conditions.

At rest, the total weight is supported by the hull buoyancy, and as the metacentric height is generally negative, lateral stability is obtained by means of wing tip floats. Owing to the high centre of thrust and low water and air resistance at speeds up to say 10 knots, the throttle is opened slowly, and elevators held up to prevent trimming by the bow. As the speed increases, elevators are put neutral when hull will gradually trim back of its own accord. From this speed onwards the load supported by buoyancy is gradually taken up by the aerodynamic water forces acting on the planing surface, the fore body rising first, followed by the tail, which may be assisted by putting the elevators down slightly until the boat is planing cleanly. During this period the water resistance increases

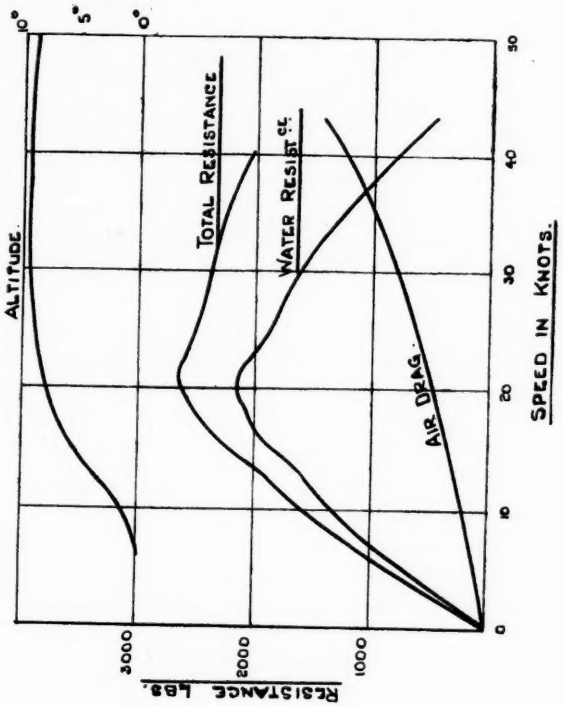

ALTITUDE.
TOTAL RESISTANCE
WATER RESIST^{CE}.
AIR DRAG.
RESISTANCE LBS.
3000
2000
1000
0
10
20
30
40
50
SPEED IN KNOTS.

FIG. 1.

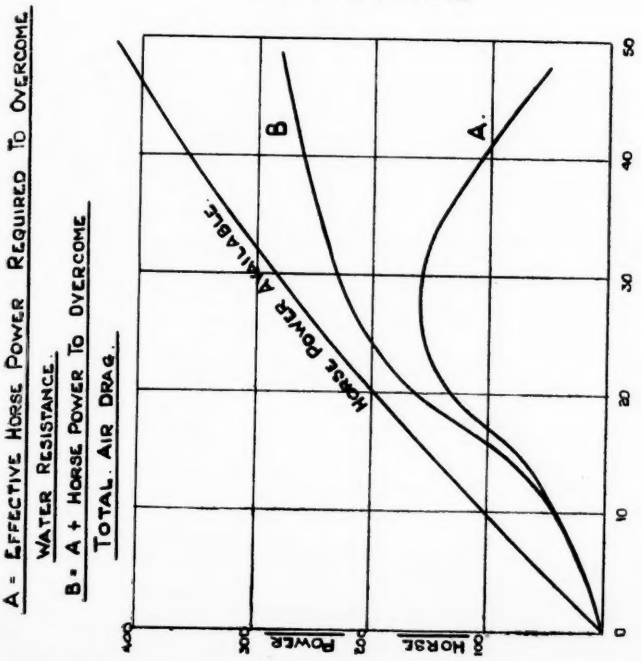

A = EFFECTIVE HORSE POWER REQUIRED TO OVERCOME WATER RESISTANCE.
B = A + HORSE POWER TO OVERCOME TOTAL AIR DRAG.
HORSE POWER AVAILABLE
B
A.
HORSE POWER
400
300
200
100
0
10
20
30
40
50
SPEED IN KNOTS.

FIG. 2.

steadily, after which if the hull is well designed it will continue to drop owing to the improved working of the step and to the increased share of the load taken by the wings. The speed at which the water resistance reaches a maximum is usually known as the " hump " speed, and is shown clearly in Fig. 1. Generally a boat which gets over the " hump " has ample power for flight. From this speed onwards the pilot has sufficient elevator control to trim the boat to any attitude giving least resistance. If there is little change of resistance with change of attitude, the boat is gradually trimmed back, accelerating all the time, until the lowest flying speed, or more commonly several knots above, is reached, when by a sharp pull up of the elevators, a clean break-away is made from the water, after which the load is entirely air borne. The elevators are then depressed before trimming to gain height.

Up to about " hump " speed lateral stability is obtained by the use of wing-tip floats and such aileron control as is available. After which speed, the boat trims naturally on an even keel due to the stable hydrodynamic forces acting on the planing surface. Also the aileron control becomes more effective as the speed increases.

The consideration of the above and of the problem of safe and easy landing leads to the following requirements, which are fundamental to success in the design of a flying boat hull.

(1) Avoidance of diving at low speeds.
(2) Seaworthiness.
(3) Hydroplaning efficiency, and landing with minimum shock.
(4) Stability at high speeds on the water.
(5) Ability to trim fore and aft to enable to take off or land.

Before discussing the influence which the requirements under the above headings have on the hull lines and dimensions, a brief description of the hulls tested and evolved at Felixstowe will now be given.

The first experiments were taken in hand early in 1915, when no engines of high power were available, hence the chief difficulty met with was the problem of taking off with a reasonable load. Thus the early experiments were made with the object of developing hydroplaning efficiency, such questions as safe landing, seaworthiness, stability, etc., were more or less neglected until more powerful engines became available. The first hull tested was a modified Curtiss " America " flying boat (Fig. 3) No. 950. Weight, light, 3,100lbs.; fully loaded, 4,500lbs.; h.p., 160; length of hull, 30ft., single step, projecting fin forward, ending at step, which was under the C.G. Fore and aft angle between the underside of the tail and planing surface of the ship was 10deg.

At high speeds all single step hulls balance on the step, and trim depends upon the angle of the tail portion, which, to avoid water drag, should be held up during the acceleration period. The original tail plane was lifting, which was excellent from this point of view, and as much load as could be flown with could be taken off the water. To improve the stability in flight, especially trim engine on and off, the tail was made negative. In smooth water there did not seem to be any appreciable loss in hydroplaning efficiency, but in rough weather, owing to the lack of buoyancy forward, she was very wet.

A new hull, No. 1230, was built in which the fins were narrower, and carried further aft, the underside of tail rounded, and of lighter construction. This hull proved to be inferior to No. 950, owing to the rounded underside of the tail portion, which caused increased suction, making it very difficult to lift out of the water in calm weather. With the different type of construction adopted there was a saving of about 300lbs., but it was considerably weaker than the previous hull, and failed after several landings.

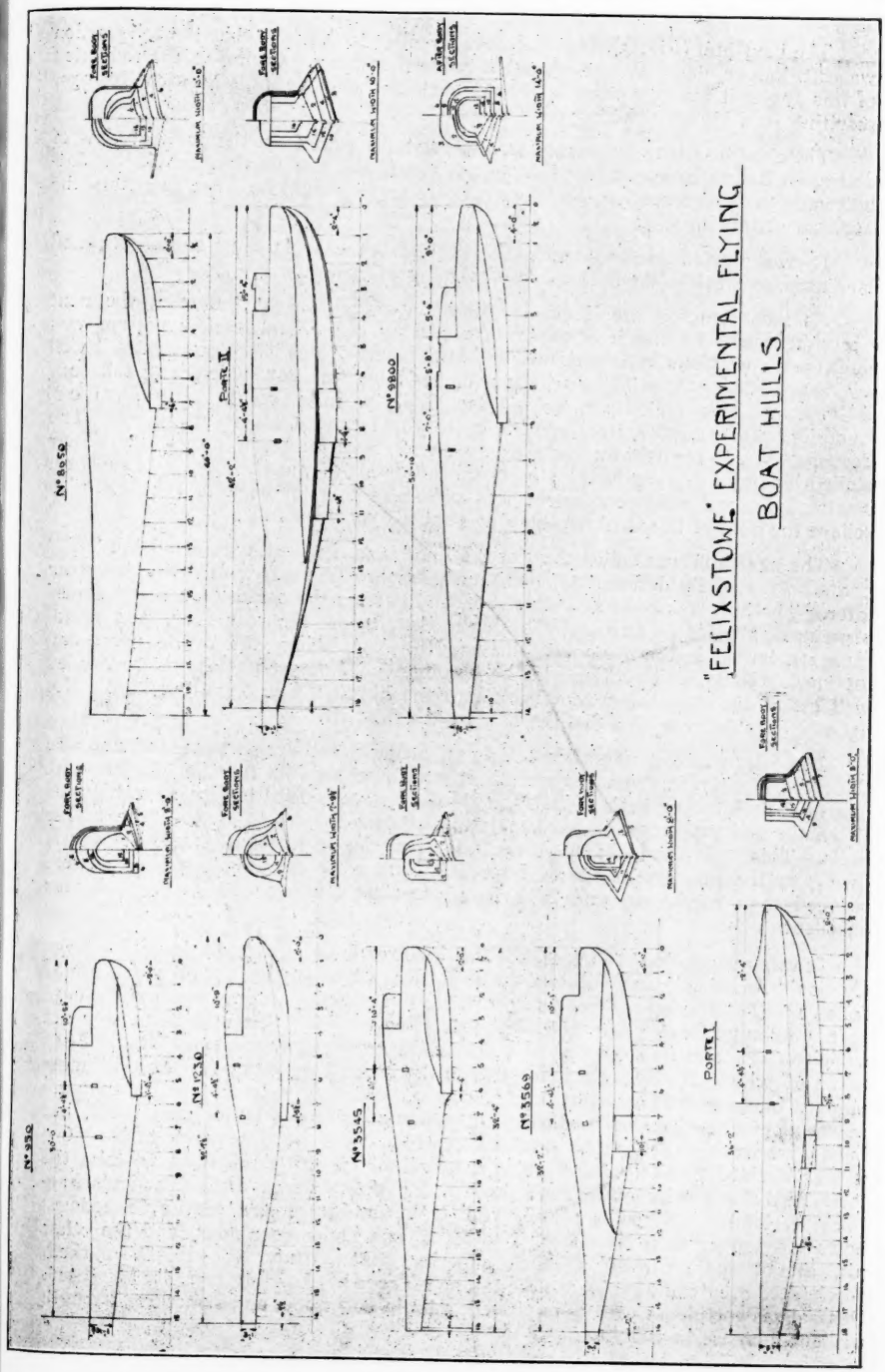

"FELIXSTOWE" EXPERIMENTAL FLYING
BOAT HULLS
No 8058
No 9800
No 3350
No 1230
No 3545
No 3560
PORT I

FIG. 3.

The next hull (Fig. 3), No. 3545, was similar to No. 950, but the tail portion was 2ft. longer, and fore and aft angle reduced to 7deg. Owing to the reduction of this angle, it was not possible to trim back to the same angle as with No. 950, resulting in a higher " taking off " speed.

The chief lesson to be learned from these hulls was, from the point of view of ability to hydroplane and low taking off speed, the tail portion should be flat-bottomed to reduce suction, and fore and aft angle should be large to obtain the necessary trim.

It was now decided to tackle the problem of easy landing conditions, and increased strength without sacrifice of planing efficiency.

As most landing breakages in those days occurred at the step, the next experiment was to find if a step was necessary. A complete new bottom was built on the previous hull, with no step, but steeper " Vee " bottom, and tail well swept up. No. 3569. The engine power available was not sufficient to take off. A step was then added 5ft. behind the C.G., when 4,200lbs. were taken off. Landing was exceptionally easy owing to the large fore and aft angle. The deeper " Vee " resulted in little or no shock, with either a normal or nearly stalled landing. Owing to the step being so far aft of the C.G. the hull ran at a small angle, and a large moment was necessary to trim back to take off. To relieve the pilot of this load, the step was shifted 3ft. forward.

The next hull was called the Porte I., and was the prototype of all the " F " boats. An entirely different type of construction was used, which will be shown later. The hull was built at Felixstowe, and carried the same " Curtiss " super-structure as before. Originally a single step, below the rear spar, was fitted. Hull 2ft. longer in the nose and tail than No. 3545, being 36ft. overall; fore and aft angle 18deg., tail portion 7in. higher than on No. 3569. Fins were carried well aft of the step and swept back into the hull. " Vee " bottom as on No. 3569. Bows fuller with a distinct flair on the first 3ft.

Difficulty was experienced in taking off owing to tail drag. A second step was therefore fitted 7½ft. from the stern. It was now possible to take off, but with less load than earlier hull. Finally, a third step was added intermediate between the main and aft steps. This brought load capacity up to that of the earlier hulls. This hull was in many respects far superior to any hull tested previously owing to the improved form of bows, cockpits were dry, landing shocks were reduced to a minimum, and behaviour generally landing and getting off was excellent.

It was now decided to experiment with larger hulls, and to begin with a large Curtiss " America " was obtained (Fig. 3), No. 8650, particulars of which are as follows:—Total weight, 8,700lbs.; twin 160 h.p. Curtiss engines; hull 40ft. long, 11ft. maximum beam; fore and aft angle 7½deg. With this load there was not sufficient power to take off. 240 h.p. Rolls-Royce engines were then fitted. Load taken off with difficulty, principally due to the lack of buoyancy forward. A most marked hump speed, about 18 knots, was noticed. At a later date, when more powerful engines became available, these boats did some quite good work, but hulls were weak structurally. As the Porte I. hull was much superior to No. 8650 it was decided to design and build a new hull on the same lines to take the " America " superstructure, and known as the " Porte II." Particulars:—Weight, loaded, 16,500lbs.; hull, 56ft. 10in. and fore and aft angle 20deg.; bows 2ft. longer than No. 8650; two steps, one under rear spar, and the other 7ft. aft of spar, proved a much superior boat. Hump speed less marked, accelerated evenly and rapidly to the taking off speed. Loading capacity increased. General seaworthiness greatly improved, gain in buoyancy, and structural strength without increase in weight.

Porte Baby No. 9800.—The building of this large experimental flying boat was carried out during the same period as the experimental modifications to the Curtiss "America" hulls were in hand, and while the experience gained from them was incorporated, it was not possible to take advantage of the results from the Porte I. Experience with these hulls confirmed the results obtained with the "America" hulls. Particulars:—Weight, loaded, 16,500lbs; hull 56ft. 10in. long; beam at step, 14ft.; width of body, 7ft.; fitted with three 250 R.R. engines; wing as tractors, and centre a pusher.

Trials showed the length of forebody was not sufficient to prevent wallowing in a following sea. Bows were lengthened 3ft., which enormously improved water performance. Air pipes fitted to the step were found to be unnecessary.

Considering the low power and the use of stranded cables throughout, the air performance was quite good. Top speed, 68 knots; climb up to 3,000ft., about 150 ft./min. Time to unstick with full load, about 35 seconds.

It was decided not to proceed further with the development of the type, as the water performance was much inferior to the Porte II., and type of hull construction weak.

Taking into consideration water performance and hull construction, the most promising type to develop was obviously the Porte II. This led to types known as the F.2A., F.3, and F.5. In the F.3 and F.5 types the hull was lengthened 3ft., otherwise the lines were essentially the same as those of the Porte II. Increased h.p. and improved detail and aerodynamic design led progressively to greater load capacity and air performance. These types were put into production and used extensively during the late war.

From the experience gained with these later types, it was decided to construct a still larger boat. The P.S.B. or Porte Super Baby, officially known as the Felixstowe Fury, was the result. Fig. 4 shows profile, plan, and body sections in detail. It was originally designed for 24,000lbs. total weight, and to be fitted with three 600 h.p. R.R. Condor engines. As these engines did not become available, five Eagle VIII.'s had to be used, which led to a decided drop in air performance.

From every point of view the boat was the best design turned out at Felixstowe. It was found that the normal load could be increased to 28,000lbs., under which loading, seaworthiness, ease of taking off and launching was superior to that of the previous F-boats. Loading tests were continued up to 33,000lbs., at which load Colonel Porte took her off in Harwich Harbour. Landing under all loads was without perceptible shock.

The extraordinary behaviour of the hull in rough seas was due mainly to the buoyancy of the hull and the lines adopted. With a load of 28,000lbs. the chine at main step was not submerged. Under all conditions, the propellers, cockpit and tail plane were clear of water thrown up. From this point of view it was found that the bow sections could be improved considerably. These were too blunt, resulting in unnecessary pounding when getting off or landing in a rough sea, or at moorings. It was decided in any future designs to drop the keel profile forward, keeping everything else the same, thus obtaining a finer entry, without loss of buoyancy forward. The net result of all these experiments is that the lines and dimensions (Fig 4) of a successful flying boat hull for a given displacement have been evolved. It now remains to show how the various features in the design contribute towards the fulfilment of the requirements laid down above, and to indicate where, if at all, these differ from what might be deduced from tank tests.

Taking the headings in the same order—

(1) At low speeds all boats have a tendency to trim forward, which is further increased in a flying boat by the high propeller thrust line, and clearly any boat

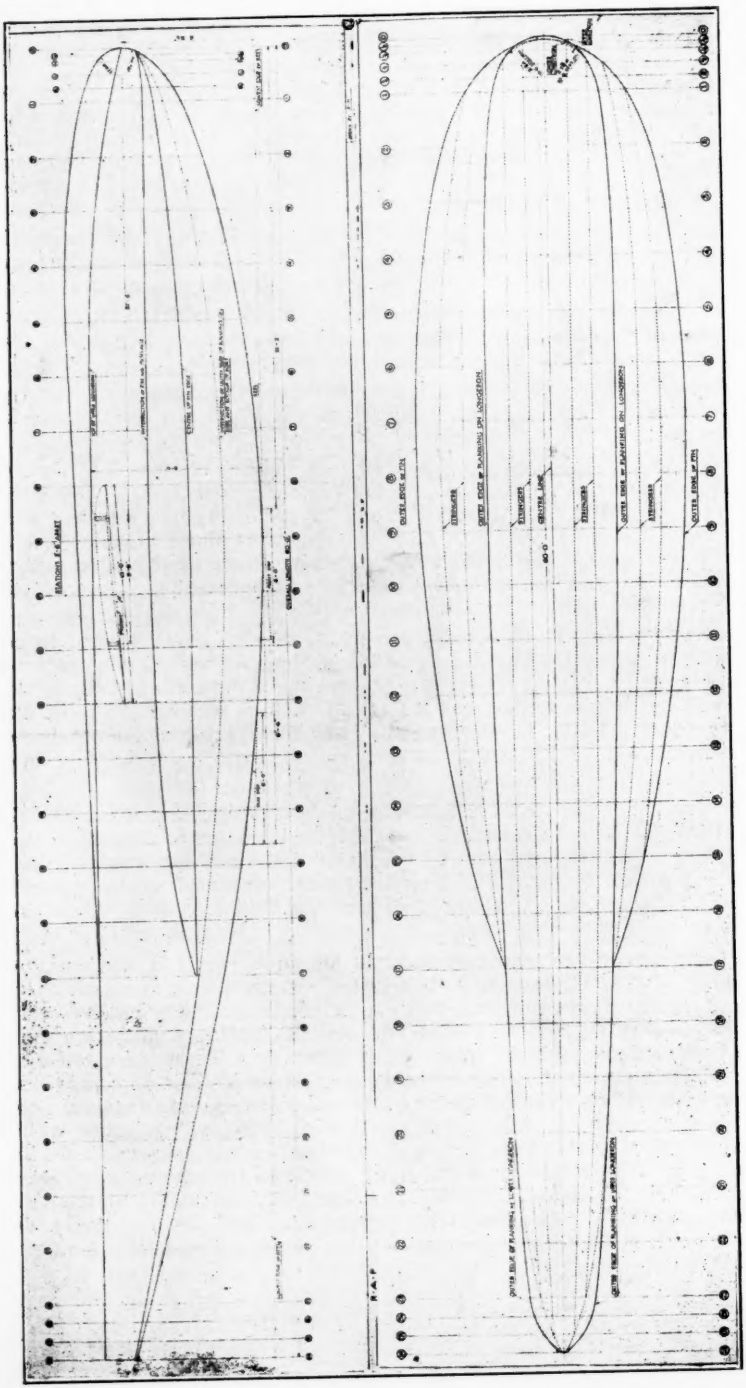


FIG. 4.

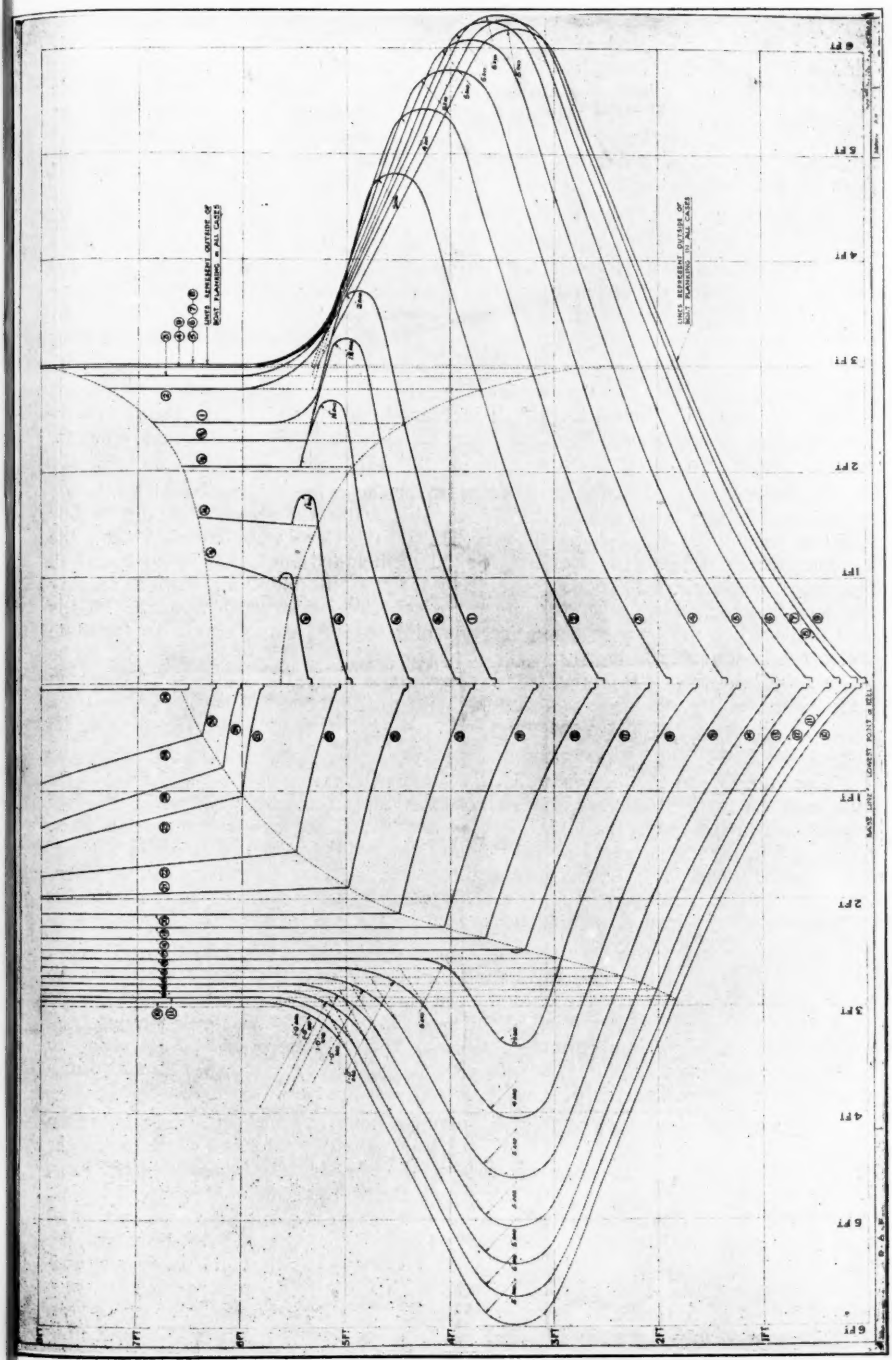


FIG. 4 (*continued*).

FIG. 4.

could be made to dive under by applying a large enough nose trimming moment. This is overcome by setting the hull surface under the chine from the main step forward at a slight positive angle about 2deg. to still water line, the angle increasing in a gradual manner up to the stem. This results in a well uplifted chine and a fairly long forebody; otherwise, should this surface attain some negative angle, called the critical angle, an area of reduced pressure will be created, drawing the bows below the water.

As the tailplane is well in the slipstream, further prevention is obtained by opening the throttle slowly, keeping the elevators well up.

(2) Seaworthiness. By this term is meant seakeeping qualities, with special regard to ability to take off, and land in smooth or moderately rough seas, keeping the cockpits dry, propellers and tailplane clear of solid water. Also, in addition, such matters as reserve buoyancy, wallowing in a following sea, and easy riding at moorings.

Clearly, other things being equal, the larger the hull the more seaworthy it will be. Increase of dimensions means increased weight and air resistance, therefore the hull should be of minimum dimensions to fulfil the above requirements, provided other considerations are of little importance.

All hulls send up and outwards from under the chine, beginning at the bows, a V-shaped blister in plan. To turn down this water the sections should be fine and flat just under the chine. The P.S.B. was better in this respect than the F-boats, which had straight sections of 150deg. included angle. A flying boat hull meeting a wave should cut it, then lift to it, without pounding, and swamping the cockpits, etc., and with minimum retardation of its forward motion. This is best achieved if the bow sections are fine and flat under the chine, to facilitate which the keel profile forward may be dropped. The buoyancy forward should be large to attain this, the chine should be full in plan, the net result being an increasing stern trimming moment with depth of immersion of the bows, and sea thrown well clear. In this respect the features which are influenced by (1) and (2) are mutually helpful.

Assuming good sections forward, the longer the forebody is, the cleaner is the hull, and the further outwards does the blister meet the wings. This is of great importance in the case of a multi-engined boat, with engines between the wings, taking off in a rough sea. From considerations of directional stability in the air, the forebody should be short to decrease the side surface forward. When, however, this dimension does not exceed .45 of the length of the hull, no trouble is likely to be experienced in this respect.

The freeboard at the lowest cockpit opening should not be less than 3ft. 6in., and the still water level line, the boat fully loaded and trimmed slightly by the stern, should not be below the chine at the main step, otherwise the boat will be very dirty and sluggish, especially in a moderately rough sea until running well on the step. While this occurs mostly during the transition period from about 10 knots until just over the hump speed, it is the ability to get over this stage in a rough sea which settles the possibility of taking off at all.

For a given displacement this will mean a larger and more beamy hull than would be found necessary from tank tests in smooth water. For example the N.4 hull (lines of which are shown in Fig. 5) of 32,000lbs. displacement, 64ft. long, 9ft. beam over chine at step, may be compared with the P.S.B. hull, at 28,000lbs. displacement, hull 60ft. long, and maximum beam 12ft. 6in. In the N.4 type of hull the fin top, until well forward, is entirely below the still load water line. The excellent behaviour of the P.S.B. hull under smooth and rough sea conditions was conclusive proof that in hulls with projecting fins the chine at step should be not more than 2in. below, and preferably above the water, when at rest. In this respect only the straight sided hull has the advantage over the projecting

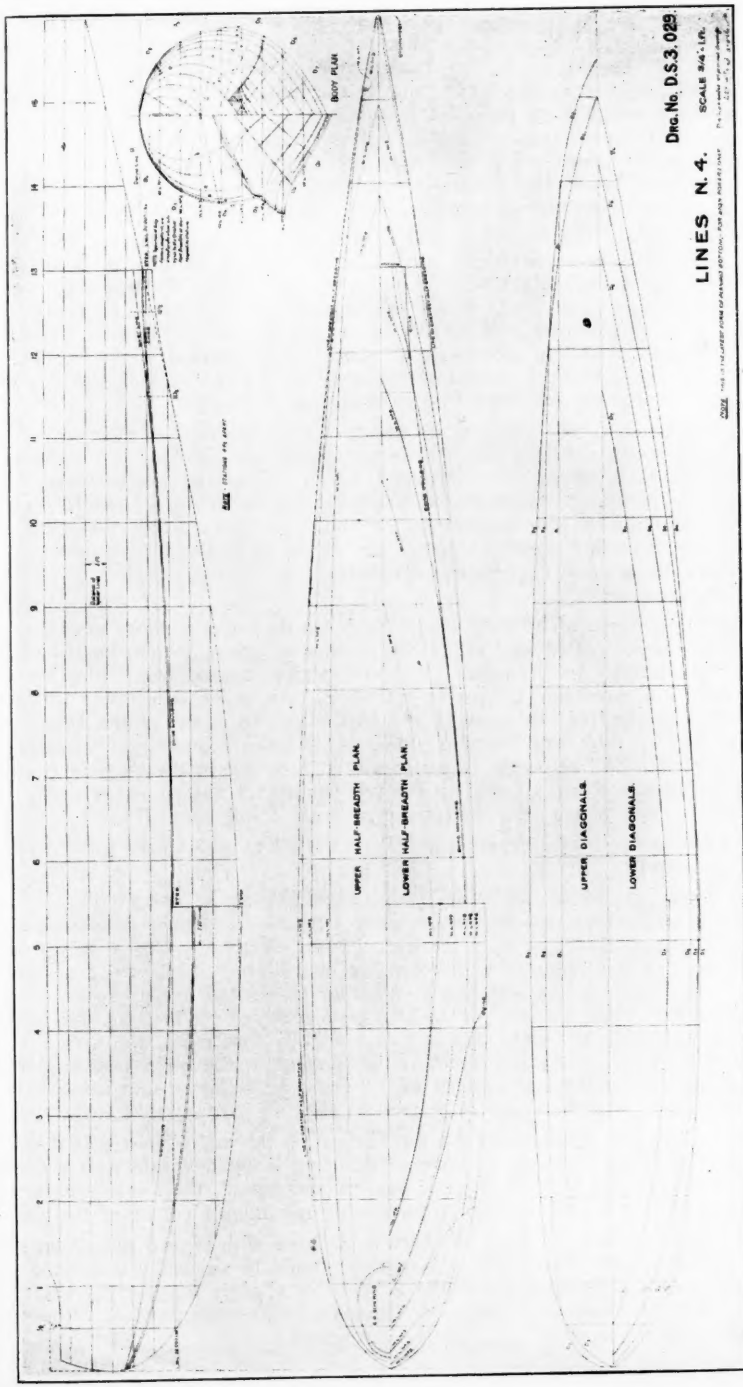

LINES N. 4.
Drg. No. D.S.3. 029.
SCALE 3/4" Ln.
BODY PLAN
UPPER HALF BREADTH PLAN.
LOWER HALF BREADTH PLAN.
UPPER DIAGONALS.
LOWER DIAGONALS.

FIG. 5.

fin type. Tank tests have shown that for a given displacement the effect of increase in all dimensions passing from a 35ft. to 46ft. hull, the maximum h.p. required is decreased about 26 per cent. at the hump speed, while the weight of hull increases roughly 75 per cent. assuming the weight of hull increases in proportion to the skin area. Also, with increase of beam only, for a given displacement at all speeds below the hump speed, the resistance is increased, but with very large beams the hump speed occurs earlier and the h.p. required is less. In all cases decrease of beam increases the maximum h.p. required.

On the other hand, considerations such as space required for cargo and passengers would probably increase the dimensions above those obtained from calculations involving only h.p. and hull weights. Also, if the Porte type of hull construction was adopted, the increase in weight would not be proportional to the skin area, as the greater part of the upper portion of the hull may be of very light construction. With good design, greater seaworthiness would be obtained and the increase of weight and air resistance of the larger hull would not be entirely a loss, from a commercial point of view at any rate.

(3) In this case the requirements are to a certain extent conflicting. Other things being equal, the boat with flat sections from keel to chine in the region of the step will take off with the largest lbs./h.p. Landing, however, would be difficult in a moderately rough sea, and at all times would require skilful piloting, to ensure landings without damage to the hull. To obtain good cushioning, the sections should be of " Vee " formation or convex as in the P.S.B. (see Fig. 4). With the high-powered engines now available, the sacrifice in planing efficiency is of little importance.

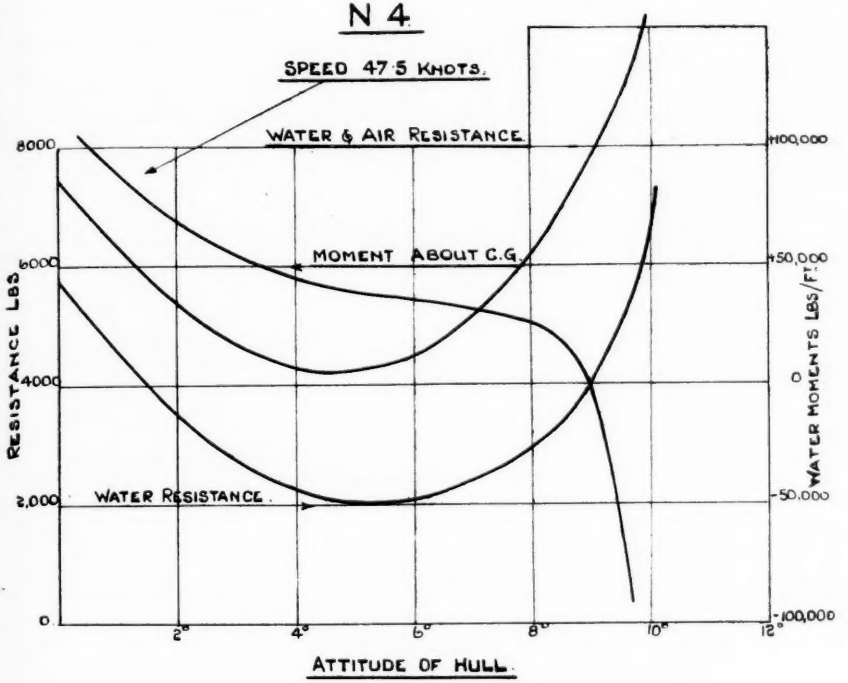
The suction area aft of step should be reduced to a minimum; a second step should therefore be fitted well aft of the main step, aft of which the tail portion should rise abruptly to the stern. The step should extend from chine to chine across the form, to ensure adequate ventilation. Air pipes were found useless as a makeshift method of dealing with the suction. The model of the P.S.B. hull was tested in the tank with the step ending at the chine rounding. The riband on the chine prevented free access of air to the step, owing to the water clinging to it. The step was then extended to the full beam over the chine, resulting in a decrease of about 30 per cent. in resistance at the hump speed.

A suitable depth of step for a 45ft. and 60ft. hull was found to be 3in. and 5¼in. respectively.

(4) From just above the hump speed to the getting off speed, that is, when the boat is on its step, the F-boats tend to porpoise, especially in a rough sea. Between these speeds there is generally sufficient elevator control to check these oscillations. In this respect the F-boats are inferior to some forms developed at the tank, in which porpoising has been either eliminated or suppressed until a high speed has been reached. This has been achieved by keeping the running angle small, placing the main step under or slightly forward of the C.G. and the two steps far apart, the angle between the tangent to the main step at the keel and the line joining the keel at step to the chine of the after step being kept as small as is consistent with ability to trim to take off.

(5) Taking any line, usually the top surface on the hull profile, as datum, the angle of the chord of wings relative to this line is fixed within a few degrees by the condition that between top and say cruising speed, the angle between this datum and the flight path should be that giving minimum air drag of the hull.

Also, bearing in mind what has been said above with regard to hull lines and diving at low speeds, the centre of buoyancy should be vertically under the C.G. when the hull is either parallel to the L.W.L. or slightly trimmed by the stern. This is easily obtained by slight modifications to underwater shape, without any sacrifice in other respects.


N 4
SPEED 47·5 KNOTS.
WATER & AIR RESISTANCE.
MOMENT ABOUT C.G.
WATER RESISTANCE.
RESISTANCE LBS.
8000
6000
4000
2,000
0
100,000
50,000
0
-50,000
-100,000
WATER MOMENTS LBS/FT.
2°
4°
6°
8°
10°
12°
ATTITUDE OF HULL.

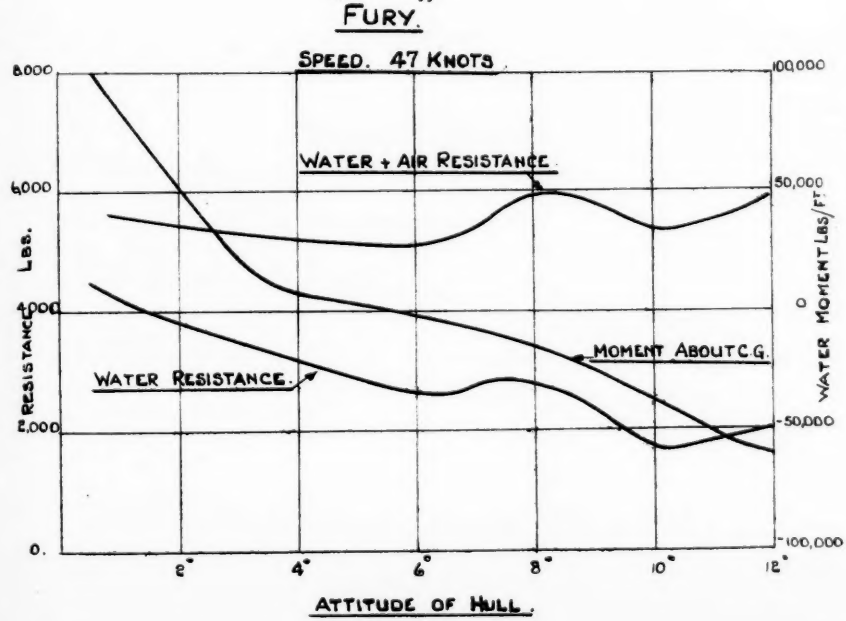

" FURY "
SPEED. 47 KNOTS.
WATER + AIR RESISTANCE.
MOMENT ABOUT C.G.
WATER RESISTANCE.
RESISTANCE LBS.
8000
6000
4000
2,000
0.
100,000
50,000
0
-50,000
-100,000
WATER MOMENT LBS/FT.
2°
4°
6°
8°
10°
12°
ATTITUDE OF HULL.

FIG. 6.

It is now necessary to be able to trim back the boat to take off to such an attitude that the angle of incidence of the wings is that corresponding to the lowest safe speed, usually a few knots above stalling speed, without excessive elevator moments and increase of hull resistance. Also, to enable a safe and easy landing to be made, it should be possible to land trimmed back on either or both steps with only a small area of the forebody, just forward of the main step, touching the water. This is attained if the fore and aft angle between the planing surface at main step and the after step is fairly large, and the tail portion, aft of the second step, swept up abruptly to the stern. This angle is best obtained from tank tests. From Figs. 4 and 5 it will be noticed that the disposition of the steps on the " Fury " N.4 hulls are different. As shown on the " Fury," are typical of all the F-boats, and on the N.4 of forms developed by Baker at the Froude Tank, N.P.L.

The former tend to run at small angles, and at high speeds may be run on either or both steps, the water resistance being least when running on the second step, trimmed well back, the change over taking place at the peak in the resistance curve. Fig. 6 shows the curve of water resistance at about six knots below taking off speed for different hull attitudes.

With the latter, the steps are far apart and the C.G. just over or aft of the main step. At all speeds both steps are in contact with the water, and there is small moment tending to keep the after step in contact with the water. If uncontrolled, these hulls run trimmed back at an angle close to that corresponding to taking off speed. Decrease of resistance is obtained by trim forward (see Fig. 6), From Fig. 6 a rough comparison can be made of the conditions under which each type is running, at or close to the taking off speed. In this connection may be given an extract from the recommendations made by the Sub-Committee on Accidents of the Advisory Committee for Aeronautics :—

" The form should be so designed that with the heaviest possible loading its natural trim (*i.e.*, running angle, hands off) shall maintain the wings at a safe flying angle."

Before reasoning from these curves it should be noticed that aerodynamic balance is assumed, which is reasonable; the water moment vanishes immediately the hull leaves the water, also allowing for slipstream effect on the tailplane, the maximum moment due to the use of full elevator movement will be of the order of 90,000lbs./ft.

Considering the case of the " Fury." The elevator control will be sufficient to trim the boat through nearly the whole range of attitude. At this speed, as will be seen from the total resistance curve, the resistance is nearly constant. At lower speeds the resistance will be less, probably at small altitudes. For this reason the boat will most likely be trimmed at a small attitude, about 4deg. to 6deg. As will be seen from the moment curve, no matter at what attitude the boat is running an air stalling moment has to be applied to trim back to "take off." Thus, unless there is ample power for acceleration or " taking off " speed, say five knots above stalling speed and elevator corrected, there is the possibility of the boat leaving the water stalled.

With the N.4 step arrangement, this condition may be made almost impossible to attain. The natural trim is about 9deg. If, therefore, the hull is so attached to the wings that the incidence at this attitude is say 3deg. below stalling incidence, which will only increase the getting off speed by about 5 per cent., then the boat can be taken off without change of trim. Any tendency to trim back to larger angles is prevented by the rapid increase of water moment. In other words, the conditions present at this attitude are stable.

The minimum resistance occurs at about 5deg., and to maintain this a nose diving moment has to be applied by the elevators; to trim back to " take off " leads again to stable conditions.

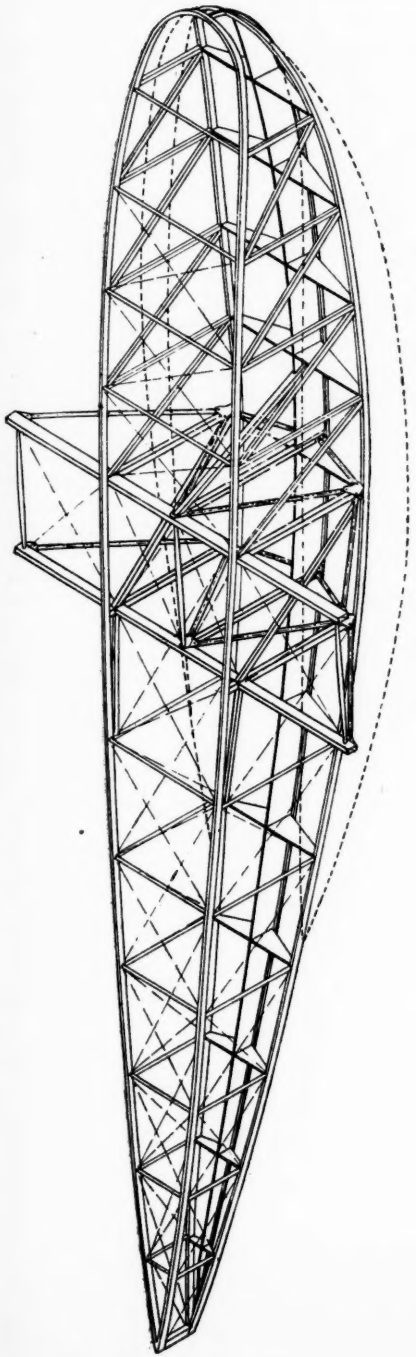


FIG. 7.

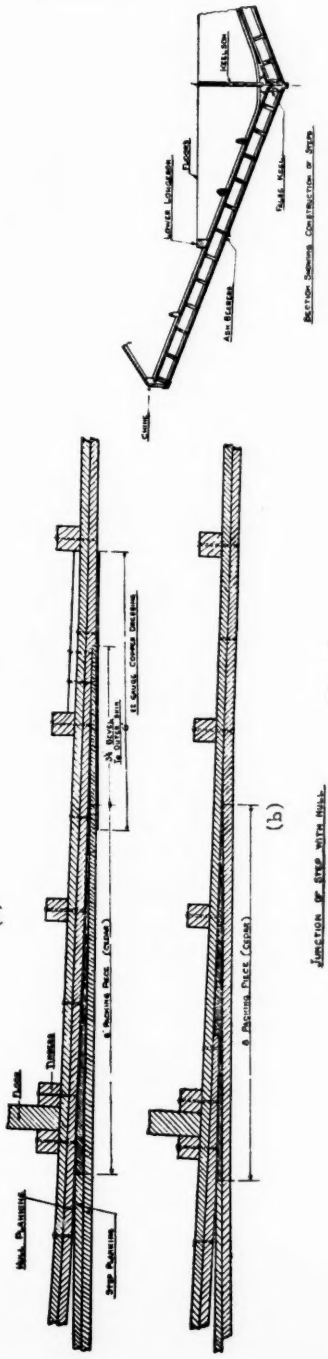


FIG. 8.

From the point of view of safety under full load, the N.4 arrangement of steps is undoubtedly the better.

On the other hand, with the exception of the " Fury," serious accidents due to " taking off " in a stalled condition are rare, as flying boats are taken off generally well above the lowest flying speed, provided sea conditions are suitable. There is, however, a temptation to " take off " nearly stalled in a rough sea accompanied by a strong wind, in order to reduce the run on the water.

There is no reason, constructional or otherwise, why the steps on the " F " boats could not be altered to retain the above stable conditions, should such be desired.

The foregoing brief discussion has been concerned principally with the hull, under water lines, and overall dimensions. With regard to the upper portion, this, as has been mentioned previously, is largely influenced by reserve buoyancy, and space for passengers, cargo, and fuel. Once the dimensions have been settled, the whole hull should be given the best practicable streamline form.

Hull Construction

The various types of construction may be divided broadly into three classes: The " Porte " or fuselage type framework; the " Linton Hope " flexible type, a development of the construction used on fast motor boats and hydroplanes; and lastly, the more or less flat-sided box type.

A short description giving the underlying principles in these types of construction will now be given. It is hoped the detail design will be seen clearly from the illustrations.

The " Porte " or Fuselage Type.—So called because the main framework of the hull is of the usual braced fuselage type. Referring to Fig. 7, which is typical of " F " boat and " Fury " construction. The framework will be seen to consist of two upper and lower longerons, meeting at the sternpost and stem. Aft of the front spar, the sides and the top are braced in the usual manner with struts and wires or tie rods. Forward of the front spar, the sides are " N " girders, the top braced as above. Between and below the level of the lower longerons run the keel and solid keelson without a break, terminating at the stem and stern post. Deep solid floors on a level with top of lower longeron join up the keel, keelson and these longerons. The chine and fin top longitudinals which sweep into the lower longerons forward and aft, now complete the framework.

Continuous timbers run from chine to chine, notched through keelson on keel level and from chine to fin top longitudinal. A timber is fastened on each side of a floor, of which there is usually one at each strut position and one between struts, and two timbers between each floor. To these timbers are fastened the planking. Several fore and aft stringers are fitted, notched out to receive the timbers to which they are fastened. On the original " F " boats the planking on the bottom was double diagonal, continuous from stem to stern, fore and aft, and to chine athwart ship, and the step planking added separately as shown in Fig. 8a. The fin tops and sides to just aft of rear spar were planked with 3-ply, aft of which the sides were covered with doped fabric, a solid mahogany washboard about 1ft deep extending along this length. The top, from aft to pilot's cockpit, was covered with doped fabric laid on fore and aft stringers, supported on formers, forward of which it was planked.

These boats were designed originally to operate from sheltered harbours, such as Felixstowe, for which they were quite suitable. When operations had to be carried out under less favourable sea conditions, several weaknesses became apparent. Water leaked into the joints on the 3-ply on the fin tops and hull sides, rotting and opening up the laminations. Similarly the wash boards split and

the fabric rotted. The fin tops were then planked double diagonal with mahogany and cedar, and the sides either planked single fore and aft and fabric covered or planked with " Consuta."

Owing to the lack of flexibility in the fin construction the joints at the chine and waist line gave trouble. Fig. 9, *b* and *e* respectively. This was overcome on the " Fury " by doing away altogether with these joints, and extending the planking round the chine. Fig. 9 *a* up to the top longeron, supported at the waist line as shown at *d*. Fig. 10 shows the arrangement. Also the skin landings at the keel as shown at *g* was found much more satisfactory than as shown at *f* (Fig. 9).

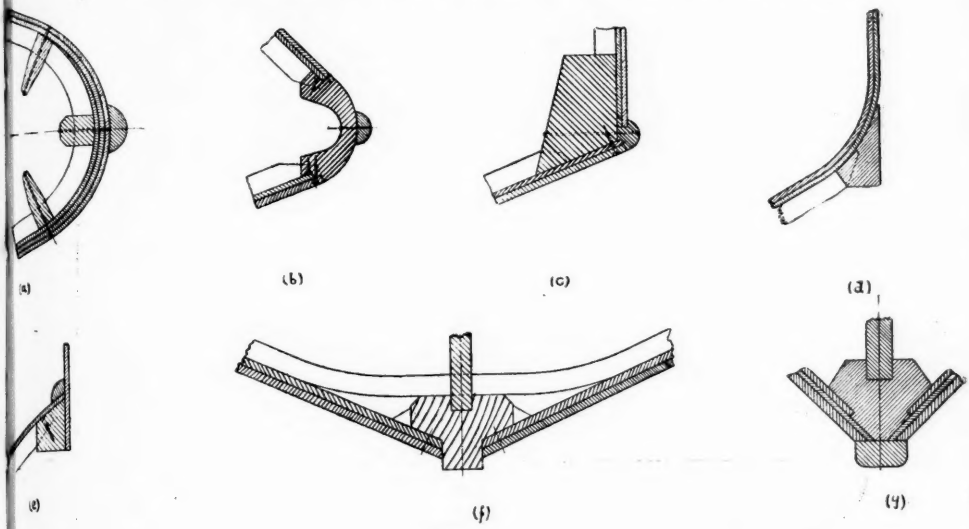


FIG. 9.

The solid transverse mahogany floors were fastened to the lower longerons by metal angle plates and notched out for two-thirds of their depth from the bottom to fit over the keelson, which was notched out one-third of its depth from the top. The corners at the joint were filled with square fillets for the whole depth. This was a weak and uneconomical form of construction, resulting in frequent splittings of the floors. On the " Fury " the keelson and floors were of built-up lattice girder construction, the top rail of the floor continuous over the top rail of keelson (Fig. 10).

Trouble was experienced with the steps, which in some cases were wrenched off the bottom. The steps were of the open type, and the planking fastened with wood screws to the bearers, which were of ash, spaced about 6in. apart and through-riveted to the hull timbers and bottom. Fig. 8 (*b*) shows the arrangement of planking whereby this trouble was eliminated. The inner hull skin is continuous from stem to stern. The outer skin continuous from stem to end of steps, forming the outer skin of the step. The outer skin of the hull bottom in the same way forming the outer skin of the after step.

These modifications were carried out in the design of the " Fury " hull. As was to be expected, she proved to be an excellent seaworthy boat, and in all ways a noteworthy advance on the " F " boat design.

Apart from the hull construction it will be noticed from Fig. 7 that the

wing roots, from the foot of engine bearer struts, with the keel and longerons form a complete structure. This is unique to the " Porte " boats.

" Linton Hope " or Flexible Type of Construction.—In this type the framework is totally different to that of the " Porte " type. It consists (Figs. 11 and 12) of a keel and keelson continuous from stem to stern post, and a large number of fore and aft stringers distributed evenly round the periphery of the hull, which

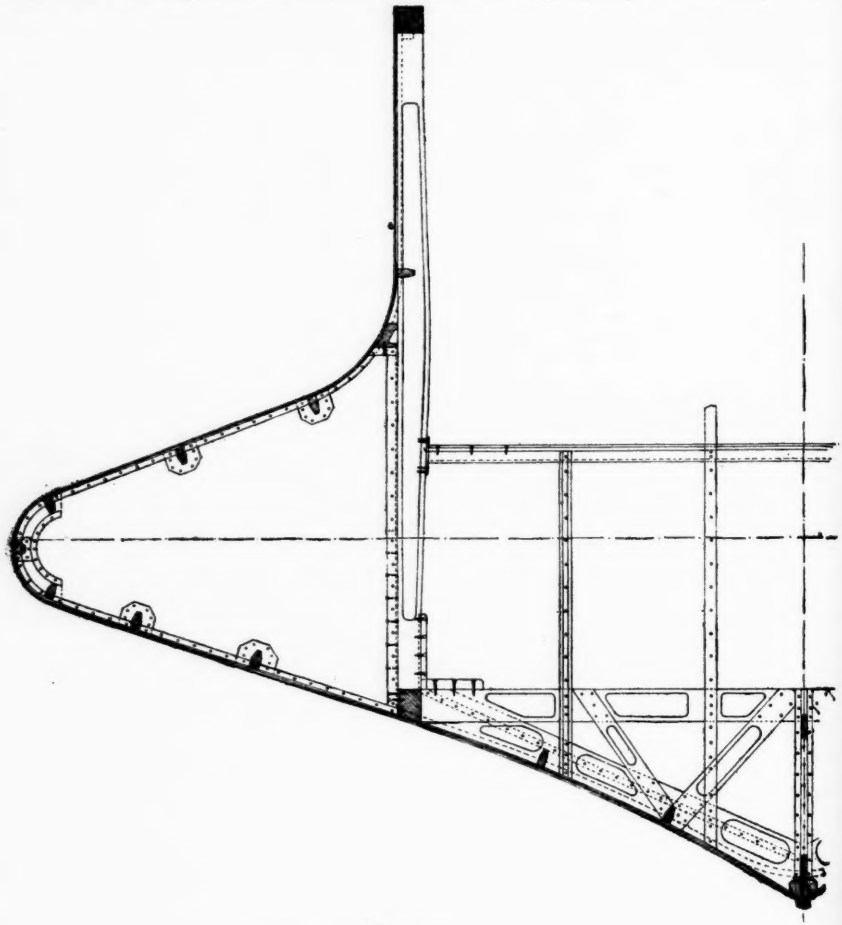


FIG. 10.

is of a streamline form. Small section timbers closely pitched are bent round these stringers, ending at the keel. To these timbers is attached the planking, of which generally the inner is diagonal and the outer fore and aft. Floors spaced at intervals varying from 8in. in forebody and tail to 4in. at the step, run round the lower half of the bottom, continuous through the keelson. At intervals of three or four feet continuous single hoops are fastened to the top of keelson and stringers. Where the wing roots spars enter the hull, double hoops, one each side, are fitted as above. This forms the hull proper. The planing surface, which

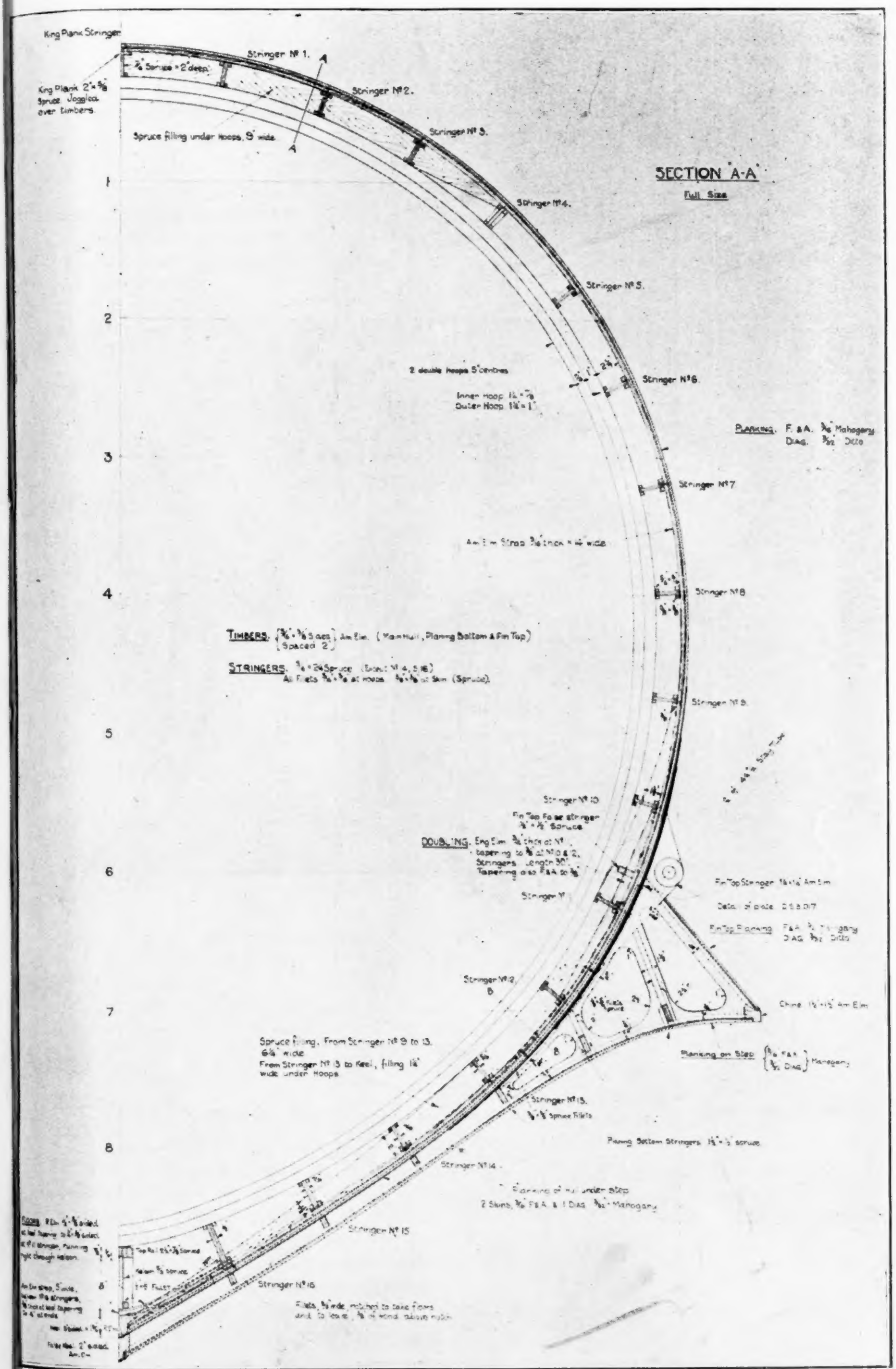

SECTION A-A
Full Size
Stringer Nº 1.
Stringer Nº 2.
Stringer Nº 3.
Stringer Nº 4.
Stringer Nº 5.
Stringer Nº 6.
Stringer Nº 7.
Stringer Nº 8.
Stringer Nº 9.
Stringer Nº 10.
Stringer Nº 11.
Stringer Nº 12.
Stringer Nº 13.
Stringer Nº 14.
Stringer Nº 15.
Stringer Nº 16.
King Plank Stringer
King Plank 2"×⅝" Spruce Joggled over Timbers
Spruce filling under Hoops, 9" wide.
2 double Hoops 5" centres.
Inner Hoop ¼"×⅞"
Outer Hoop ⅛"×1"
PLANKING. F. & A. ⅜" Mahogany Diag. ⅜" Oute
TIMBERS. {⅝"×⅝" Sides} Am. Elm. (Keelson, Planing Bottom & Fin Top) {Spaced 2"}
STRINGERS. ⅞"×2½" Spruce (Bows Nº 1 & 2 ⅝"×1½") All Fillets ⅝"×⅝" at Hoops ½"×⅝" at Skin (Spruce).
Am. Elm Strap ⅜" thick × 4" wide
DOUBLING. Ang. Elm ⅜" thick at Nº 1 tapering to ⅛" at Nº 4 & 12. Stringers Length 30" Tapering one F & A to ⅛"
Fin Top Fore & Stringer ⅜"×⅝" Spruce
Fin Top Stringer ⅝"×1½" Am. Elm
Fin Top Planking F & A ⅜" Mahogany Diag. ⅜" Oute
Chine ⅝"×1½" Am. Elm
Planking on Step {⅜" F & A ⅜" Diag.} Mahogany
Spruce Filling. From Stringer Nº 9 to 13. 6½" wide. From Stringer Nº 13 to Keel, filling 14" wide under Hoops
Planing Bottom Stringers ⅝"×⅝" spruce.
Planking of Hull under Step 2 Skins ⅜" F & A & 1 Diag. ⅜" Mahogany
Fillets ⅝" wide, notched to take Floors and 3 rivets. ⅜" at Hoop. Above Hoop

FIG. 11.

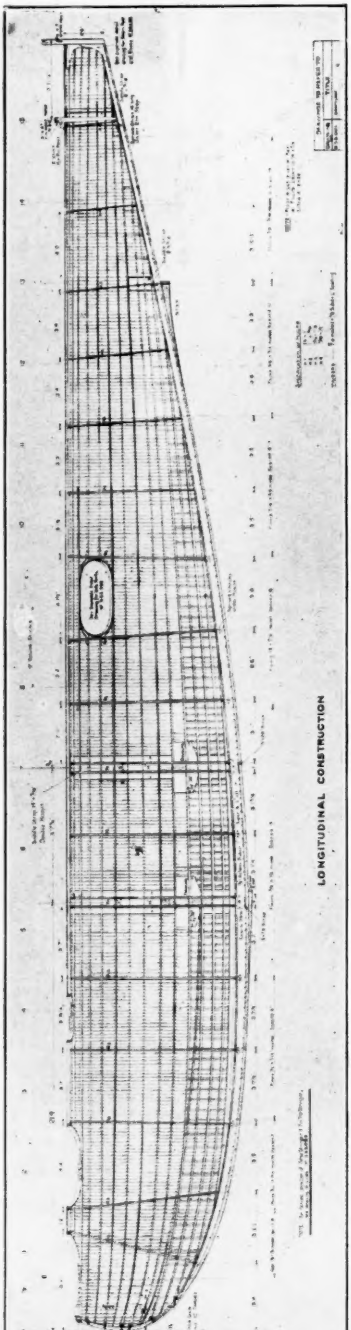


FIG. 12.

extends from the bows to the main step, is attached to 3-ply formers on the hull bottom, and at the step, which is closed, to fore and aft bearers. The fin top is straight and the chine ends at the step. The after step is attached in a similar manner. This construction leads to a double bottom in way of the fins.

In later developments of this type the chine is continuous from stem to after step, as in the " Porte " boats. The planing surface is continuous from stem to after step, at all points kept well clear of the hull, thus forming with the hull and fin tops a complete double bottom. The 3-ply formers are replaced by bent hoops and the top fin curved as shown in Fig. 13. The steps are open, and planking carried out in the same way as in the " Porte " boats, thus avoiding external joints.

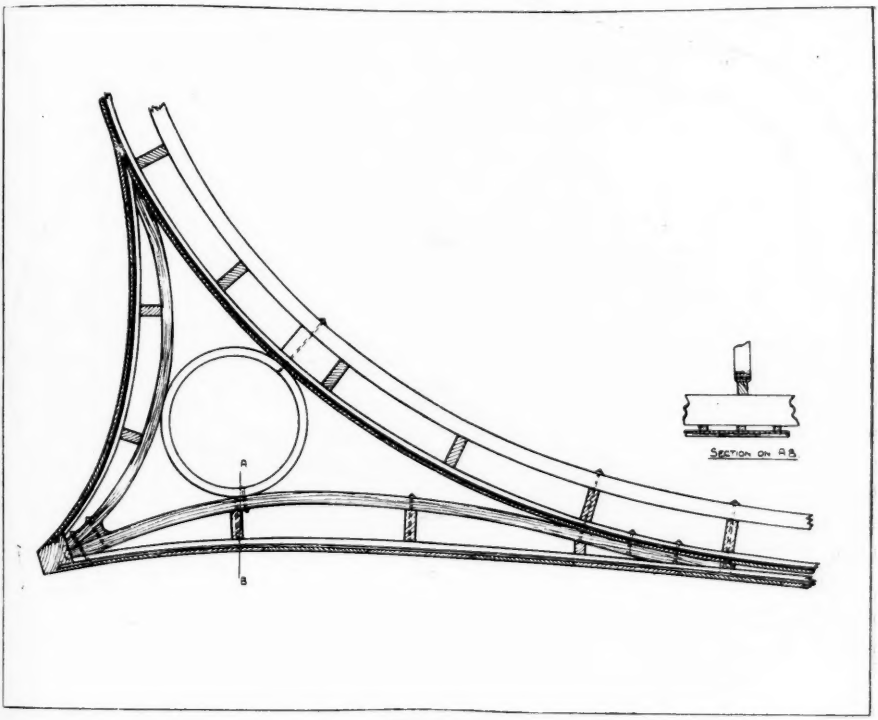


FIG. 13.

It will be noticed there is no similar wing root structure, as in the " Porte " boats.

A further example of a hull following usual boat practice construction is shown in Fig. 14, from which it will be seen the fore and aft stringers are closely spaced, and there is practically no keel and a very light keelson. The timbers are more openly spaced, and in way of and above the floors, run on the inside of the stringers. At intervals there are 3-ply continuous web frames, between which there are 3-ply web floors, stabilised with intercostals as shown. The planking at the bottom is not jointed at the keels, but runs from chine to chine. In this type also there is no wing root structure.

The "Vickers" or Box Type of Construction is shown in Fig. 15, from which it will be seen there are no projecting fins as on the "Porte" or "Linton Hope" type hulls. The framework consists of top and bottom longerons,

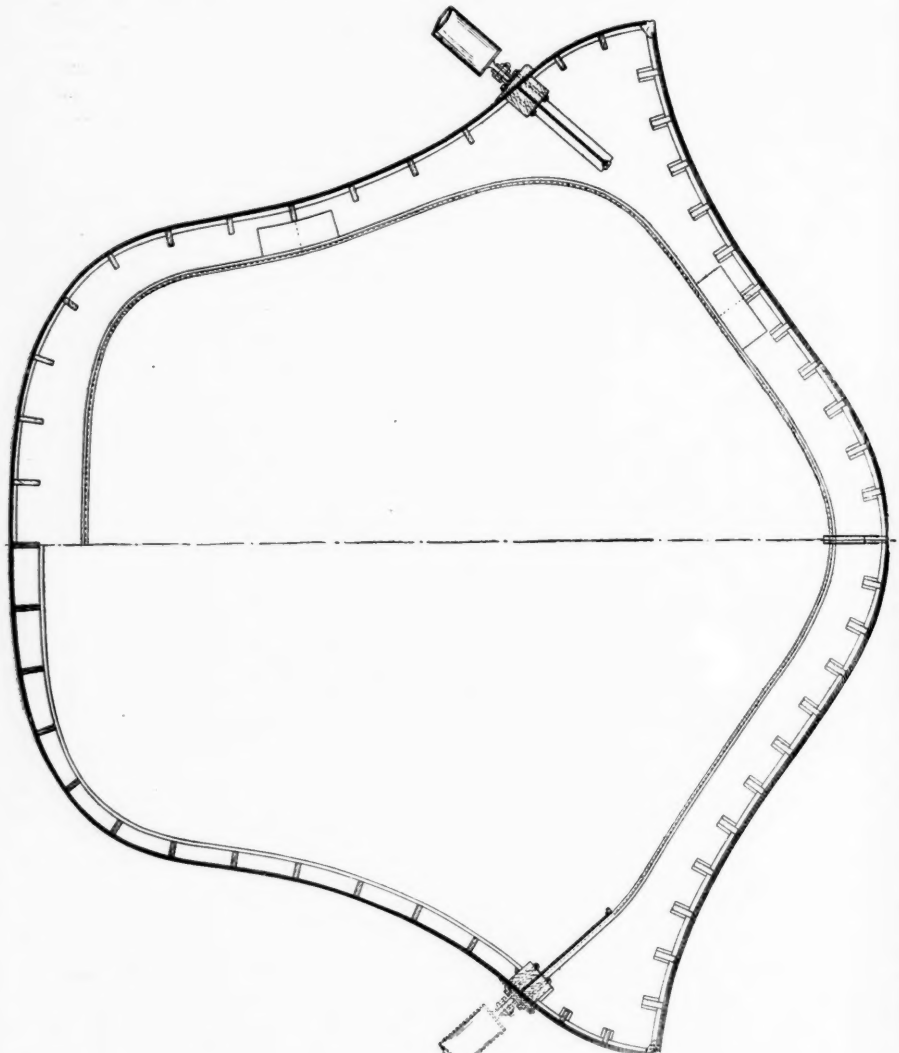


FIG. 14.

a keel and transverse box-shaped diaphragms at intervals of 3ft. Between the diaphragms are two floors, four heavy fore and aft stringers run over these floors and on top side of bottom rail of diaphragms. Timbers run fore and aft, under and fastened to these floors and diaphragms, to which is fastened the

"Consuta" sewn planking. The sides and top, which are flat surfaces, are also covered with "Consuta."

In all these types of hulls, several bulkheads are usually fitted. In the smaller boats, with the exception of, at the most, two complete bulkheads in the tail portion, it is not generally possible on the score of accessibility and weight to fit other than transverse dwarf bulkheads. Of the latter, two should straddle the main step; the disposition of the remainder, if any, will depend upon the internal arrangement of the hull. These are sufficient to limit the travel of any water in the hull, due either to abnormal leakage or slight damage to the bottom, provided such water can be kept under by the bilge pump. In large passenger boats, where safety will be the first consideration, the passenger accommodation will be located between complete bulkheads, fitted with watertight doors.

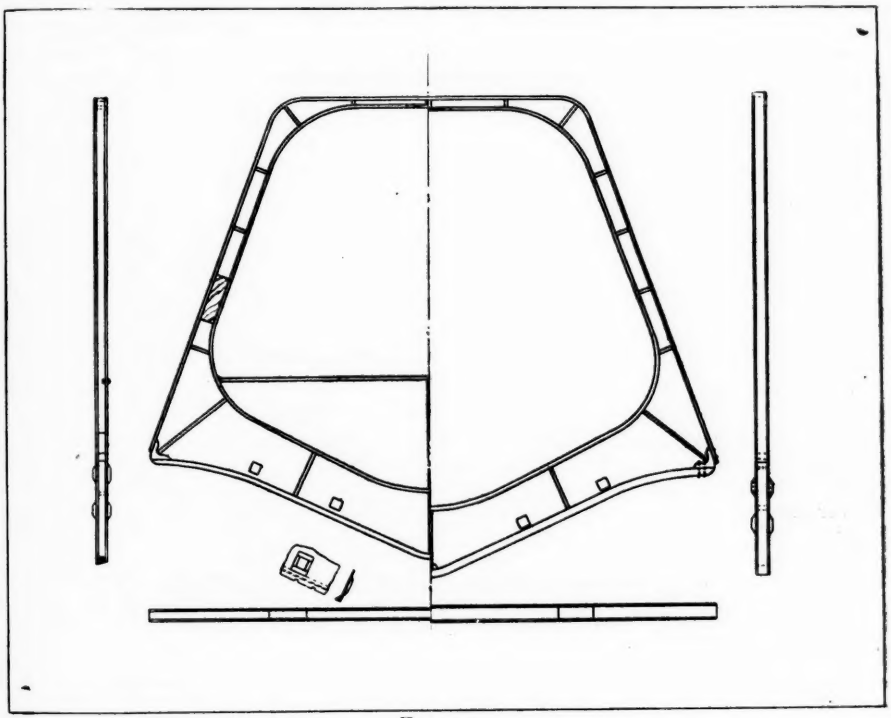


FIG. 15.

In the event of a serious crash, any form of bulkhead would most likely give way against the inrush of water, but, depending upon the extent of the damage sustained, may so reduce the rate of flooding to enable the passengers and crew to get clear.

Unless of flexible construction, the joint between the bulkhead and any part of the hull, which in itself possesses a certain amount of flexibility, should not be rigid, but should be capable of "giving" to the working of the hull. This may be arranged for either by allowing the bulkhead to slide between two faces round its periphery or by interposing a strip of strong Willesden canvas between the bulkhead and the hull shell. Bulkheads may be either of solid construction

or built up of Willesden canvas attached to a spruce framework, a form combining strength and lightness (Fig. 10).

As a further safeguard against flooding, due to extensive under water damage, double bottoms are unquestionably of great value, and in all types of construction should be fitted in way of the passenger accommodation. However, until boats of relatively much greater displacements are contemplated, it would seem unnecessary to have double bottoms extending along the whole L.W.L., as apart from the inevitable increase in weight, the majority of hull breakages occur at the step, the construction of which usually forms a shallow double bottom, and the evidence of serious failures of single bottom hulls is insufficient to insist upon their adoption.

Before discussing the various types of construction outlined above, a few remarks on the question of hull resilience would not be out of place. The greatest loads on the planing surface arise either when " taking off " or landing in a rough sea. Now these loads act for a very small interval of time, normally, in either case, less than 30 seconds. It is therefore preferable to allow the hull to " give " a certain amount, rather than to provide sufficient strength to rigidly withstand these loads. Otherwise the scantlings required would mean a prohibitive increase in weight. Further, a more or less flexible structure absorbs and dissipates energy due to shocks without imposing severe local stresses in any of the members. As the required degree of flexibility is beyond calculation, successful hulls provide the only source of information; bearing in mind the well-established fact that where a flexible joins a relatively rigid portion of a structure, the change of section should be gradual and not abrupt. In other words, discontinuity of section should be avoided.

It is now proposed to give a brief outline of the main features in the construction of each type described above.

" Porte " Type

(1) The hull framework, wing and tail plane structure, forms a complete and simple braced structure, the overall strength of which is independent of the hull skin. Whether resting on a trolley or in a dry dock, the resultant weight is transmitted to the keel and bottom longerons; at the wing roots, where it is supported, no other external support is required. No loads are taken by the hull planking, thus avoiding distortion and increased liability of leakage. In flight the weight of hull, passenger cargo, and fuel is transmitted to the point of attachment of the lift wires at the wing roots, through the framework, alone. The only drawback to the wing root structure is the absence of a clear passage through the hull.

The tail setting remains sensibly constant. In the earlier boat-built types, the tail portion sagged under load, due to skin soakage, and the effect of a possible variation in temperature between the top and bottom surfaces. With adjustable tail planes, this is of less importance.

(2) The required degree of strength and flexibility is obtained with an acceptable weight.

(3) As the overall strength of the hull is independent of the skin, cockpits, port holes, etc., present no difficulties and require little compensation.

(4) Over the greater part of the under water surface the only external joint of the skins is at the keel, at which position experience has shown least trouble is to be expected.

(5) The interior of the hull is well lighted, owing to the doped fabric top.

(6) Above the top of the fin, the sides are flat, but if planked with two skins

are amply strong to withstand any seas likely to be met, which would not otherwise seriously damage the boat.

(7) A considerable number of metal fittings, steel or duralumin, are used in the construction. These present no real drawbacks. They are readily inspected, and do not corrode or rust away in a few weeks as some critics would have us believe. With little attention they will outlive the hull.

(8) This type of construction does not readily admit of the fitting of a double bottom.

Linton-Hope, or Flexible Type

(1) Actually these hulls are flexible only in parts. The planing surface ends at the main step, which is closed, forming a rigid belt extending across the form from chine to chine, thus introducing a sharp discontinuity of section, resulting in serious local weakness. Further, the whole planing surface and part of the inner shell to which it is attached, forms a rigid girder, an abrupt discontinuity of section occurring at the waist line. In the latest types the chine is continuous from stem to after step, and the steps open, following the " Porte " type practice.

(2) Three-ply formers are used between the inner shell and planing surface. These break up rapidly under the action of water. Also, where the planing surface washes into the inner shell in the region of the keel, a pocket is formed which cannot be drained. These defects have been more or less eliminated in the latest hulls.

(3) There is no wing root structure as in the " Porte " hulls. On a trolley, or in dry dock, the weight of the wing structure, engines and fuel, if carried on the wings, has to be supported at the engine bearer struts if distortion of the hull is to be avoided. This leads to great inconvenience from the handling point of view, and in other respects may be compared with the " Porte " type under (1).

(4) Possibly of better streamline form than the " Porte " type.

(5) No metal fittings used in the construction.

(6) Lends itself readily to the fitting of a double bottom.

(7) The overall strength depends on continuity of stringers and shell. Openings in shell greatly weaken the hull unless heavily compensated.

(8) Interior of hull very dark.

(9) Skins on the planing surface are jointed at keel and chine, making four more joints than on the " Porte " type.

(10) The curvature of the hull sides and top provides greater protection from damage due to heavy seas breaking on the hull than the flat-sided type does.

The second type of hull of the motor boat or yacht form of construction calls for little comment. The hull is of a weak shape and the keel and keelson, as shown, are of inadequate strength. Three-ply frames and floors are useless. There is no wing root structure. It is a typical example of a design produced by a boat builder possessing no knowledge of flying boat hull requirements.

The Vickers, or Box Construction.

(1) The shape is simple, therefore the cost of construction should be relatively low.

(2) Owing to the absence of projecting fins the air resistance should compare favourably with the other types.

(3) The top longerons are much too light where most strength is required.

(4) No keelson or floors are fitted as in " Porte " type. Panels between

XUM

diaphragm are braced with "consuta." Strength depends entirely upon the joints at the longerons. Compensation for openings presents great difficulty.

(5) Owing to the lack of flexibility, to obtain the necessary strength, the weight will be relatively great.

(6) The internal wing root structure is of little use owing to the poor design of the hull framework.

The two latter types of construction would seem to possess all the disadvantages and none of the advantages of the "Linton Hope" or "Porte" types respectively. Sufficient experience with the "Linton Hope" type of construction has not yet been obtained to enable a fair and definite comparison to be made with the "Porte" type. As both types possess features of distinct merit, a suitable combination of these may ultimately lead to a type of construction superior to either.

Hull Weights

It has been customary to adopt as a criterion for the comparison of different types of hulls the percentage weight of the gross weight. This is inadmissible unless the types compared are of equal water performance under the same sea and weather conditions, and of equal strength and durability. Further, such items as wing roots, bulkheads, flooring and seats, having no direct bearing on the type of construction, should be omitted.

Fig. 16 gives in diagrammatic form the detail weights of the F.5 and P.5 flying boat hulls. As will be remembered the F.5 is of the fuselage type of construction, whereas the P.5 is of the Linton-Hope type. The hulls are of the same length, 45ft., and of approximately the same displacement. Both flying boats were constructed to conform to the same specification.

It will be noticed that the weight bare of the F.5 hull is considerably more than that of the P.5. This difference may be accounted for mostly by the larger beam of the F.5, in this instance about 33 per cent. greater than the P.5.

Comparing the percentage weights of the two main items of each type—the framework and skin, these are the same to all intents and purposes. Now the design of these two hulls differs in the position of the load waterline for reasons explained under Hull Lines and Dimensions. Should, therefore, it be found from experience with the Linton Hope types of hulls, as it probably will be, that the load water line is too high for rough sea work, then it may be found the additional weight involved in lowering the low water line the necessary amount would make this difference negligible.

In other words, it is not proved that the "Linton Hope" type of construction is lighter than the "Porte" type, as is generally stated and accepted. Increased air performance can always be obtained by flimsy hull construction. Until the P.5 type, both as regards construction and dimensions for a given displacement, has been proved equal, under the same conditions, ashore and afloat, to the "Porte" type, arguments based on percentage weights are misleading.

Curve Fig. 17 shows the weight of "Porte" type hulls actually constructed, plotted against the length, from which it will be seen that the weight of hulls between 20ft. and 40ft. is nearly proportional to the length, after which and over the range covered the weights vary roughly as the square.

For preliminary design purposes, a rough rule for the weight in lbs. of this type of hull, including bulkheads, wing roots, is given by the square of the length in feet.

HULL WEIGHT ANALYSIS.

F.5. (BARE HULL).

1555 lbs. (100%)

Frame Work.	Skin.		Hood.	Fabric	Sundries.
724 (46.6%)	563 (36.2%)		50 (3.2%)	40 (2.6%)	177 (11.4%)
	Sides	Bottom			Fastenings.
	130	Inner 158			Varnish
		Outer 190			Glue
		Steps 85			Paint
		433			

Longerons	158	Bracing.		Stringers and Timbers.		Metal Fittings.
Keel ...	44	Nose ...	36	Stringers ...	29.5	147
Keelson ...	29	Wing Roots	20	Floors ...	55	
Sternpost	5		—	Fin Timbers &		
	236		56	Bottom Ribs	200	
					284.5	

P.5. (BARE HULL).

1230 lbs. (100%).

Frame Work.	Skin.		Fastenings.		
560 (45.6%)	456 (37%)		103 (8.4%)	66 (5.4%)	44 (3.8%)
			Inner Hull 77	Varnish	Sheet Brass 8
	Inner Hull	After Step	Planing Surface 24	Paint	Metal Fittings 25
	Fore Planing		After Step 2	Glue	Fabric 11
	316 Surface	12	—	Sundries	—
	128		103		44

Inner Hull		Fore Planing Surface.		After Step.	
Keel and Girders	60	Timbers ...	21	Chine and Mouldings	6
Timbers ...	89	Frames ...	8	Step	4
Floors ...	30	Stringers ...	21	Sundries	2
Stringers ...	131	Chine ...	26		—
Stem	3	Step ...	15		12
Hoops	60		—		
Saddles ...	29		91		
Chocks ...					
Coambings ...	55				
Sternpost ...					
	457				

FIG. 16.

Hull Stresses

If the value and disposition of the maximum water forces to which a hull was subjected, under the worst conditions likely to occur during normal operations, were known, simple calculations could be made to ensure adequate over-all strength. Such forces however are not known, even approximately.

The worst loads likely to occur when at rest or taxi-ing at low speeds in a rough sea may be calculated, assuming the boat is supported on a wave crest at each end and the wave hollow touching the keel amidships. The buoyancy

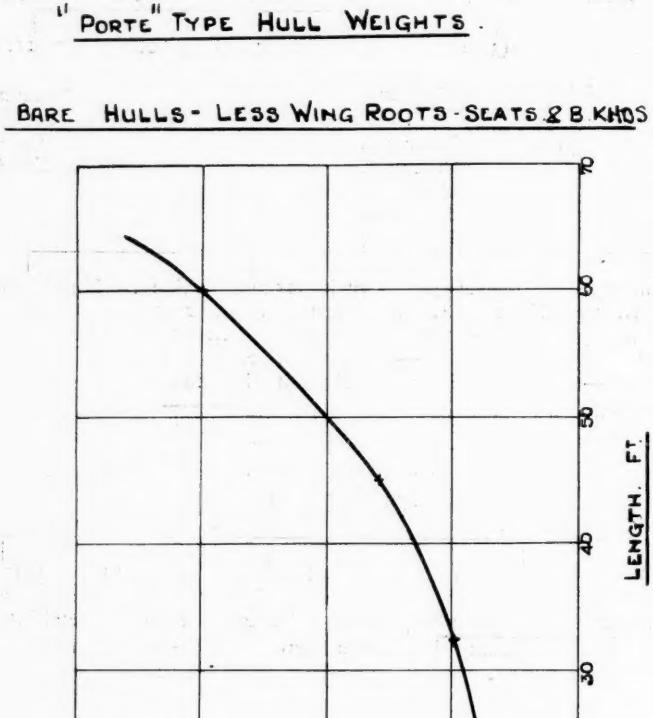


FIG. 17.

and static load curve may then be drawn from the known distribution of the various weights and the bending moments and shear forces obtained.

Unfortunately, almost identical conditions may arise when the boat bounces from one wave to another, the result of leaving the water more or less stalled, when pitching in a rough sea. In this case an estimate of the water forces does not seem to be possible, thus rendering the above method of calculation of little use.

On the other hand, an analysis, on the above lines, of hulls of proved strength might lead to figures useful as a basis of comparison.

When considering a hull from the strength point of view, the important fact should not be overlooked that a flying boat is intended to fly, and to justify its existence as a flying machine from either a military or commercial standpoint, must in addition carry a reasonably useful load. A suitable compromise has therefore to be made between weight and allowable strength. In this respect a flying boat hull is analogous to the landing chassis of an aeroplane. Experience has shown there is no trouble in obtaining sufficient over-all strength for an acceptable weight, and with good design the latter need not exceed 12 per cent. of the gross weight. With the exception of checking the tail portion for the maximum air load likely to occur on the tail plane, no strength calculations were attempted on the "Porte" hulls. Once the type of construction was settled, the necessary strength was eventually obtained by a process of elimination of weaknesses and refinement in detail design.

The fixing up of the scantlings for hulls of other displacements is then a comparatively simple matter, in which the elementary principles involved in the comparison of similar structures give considerable guidance.

All-Metal and Composite Hulls

Hulls may be constructed either entirely of wood or metal or in varying proportions, a combination of these materials. Up to at least 100ft. long a rough calculation shows that for the same strength, the weight of a steel hull would be considerably greater than the corresponding wood or composite built hull. Further, the cost of an experimental all-metal hull would be probably five times and even on a production basis would not likely be below twice that of a similar wood hull.

In the case of all steel hulls, the main disadvantages arise because of the following facts:—

(1) To keep down the weight, the frames and plating would require to be of small gauge of the order of 20 to 22. To prevent local weakness, such as buckling, the plates would need to be corrugated, the rolling and forming of which to the required shape would be very costly.

(2) Unless rusting and corrosion could be prevented the scantlings would have to be increased beyond that necessary for strength.

(3) As it is not possible to caulk the joints in such thin gauge steel plate they would require to be welded, and left in the unannealed state, thus leading to local weakness.

(4) In the event of damage to the hull, repairs would be more difficult, costly and take much longer to execute.

The only apparent gain would be the elimination of hull soakage and possibly abnormal leakage resulting from the planking opening up in tropical countries.

It would seem, therefore, that unless under circumstances which so far have not arisen, the resulting disadvantages are too overwhelming to warrant the use of steel.

By the use of duralumin the scantlings could be increased, and without sacrificing strength the weight might be slightly reduced. Apart from this, and the likelihood that the sea water corrosion would be of little importance, provided the material was properly worked, heat treated and protected by a covering of bituminous varnish or non-metallic paint, a duralumin hull would suffer from the same disadvantage as outlined above.

With a composite hull of "Porte" type construction, when the frame, keelson and floors are of duralumin and the skin wood, as at present, a consider-

able gain in strength and reduction in weight could be obtained at an acceptable increase in cost.

Wing Tip Floats

Unless the hull is of relatively great beam or fitted with extensions in the form of a parallel or tapered thick wing section, added to the chine in the region of the main step, the metacentric height will be negative. Under these conditions the boat, when at rest on the water generally, will not remain on an even keel, but will heel over to port or starboard until the capsizing moment is balanced by the moment due to water, displaced by the float, when equilibrium will be established.

The " Porte Baby " and to a lesser extent the " Fury " remained usually on an even keel when head to wind, but the equilibrium was more or less neutral, a small disturbance bringing on the conditions as above.

The requirements are simple and easily provided for in the design and are as follows:—

(1) When at rest on the water the wing tips should be clear of moderate waves, say 3ft. to 4ft. high. There is really no reason why the wing tips should extend beyond the wing tip floats.

(2) With a cross wind, same as above.

(3) Should be just clear of the water when planing at high speeds.

All the data is easily obtainable, when the necessary float displacement and water line will be fixed.

With regard to the best form for the float; experience showed that the hydroplaning efficiency should be low. It should cut through the waves with small lift and when the boat is rolling should enter the water with as little shock as possible. The plan view should be of streamline shape with pointed bows to reduce air drag. The sides flat. The keel profile more or less parallel to the wing chord and slightly rounded into a straight stem. The chine should start from the foot of the stern post and rise gradually at first, then at the bows swept well up to the stem. This will allow of fine sections forward. The sections throughout may be either convex and fairly flat at the chine or of " Vee " formation. On the larger boats it has always been found possible to attach the floats directly to the wing spars, as within limits the distance of the lower plane from the water does not increase with the size of boat. This is preferable to suspending them by a complicated system of struts and bracing.

The construction is simple and may consist of a keel, two chine and top members, stem and stern post. Several floors and side frames, two of which at least should form three-ply bulkheads, diagonally stiffened. Timber running across the keel from chine to chine and from chine to top members. The sides may be single fore and aft planking covered with fabric, and the bottom either double diagonal, or inner, diagonal and outer fore and aft planking. As the float top is snug up against the lower surface of the wing, fabric covering is sufficient. Finally drain plugs should be fitted to each compartment.

With regard to weight, Mr. A. Thom has given the following formula, which represents the facts fairly well. If W is the weight in lbs. and L the length in feet, then $W = 0.36\ L^{2.5}$.

Water Rudders

On a single-engine flying boat, especially where there is a marked turning tendency due to slipstream effect on the fin and rudder, a water rudder is almost a necessity for manœuvring at slow speeds. To be really of much use, the area

required would seem to be much larger than warranted by the extra air resistance and weight involved. At high speeds the air rudder is the more effective.

With flying boats, twin engine or more, and outboard propellers, water rudders are quite unnecessary, as the air propeller turning movement is greatly in excess of that obtained from a water propeller of reasonable dimensions. Moreover, in any commercial service it would be unnecessary for a large flying boat to be able to manœuvre at slow speeds in restricted water, as the motor-boat tender would then take control, analogous to the case of the large steamship entering or leaving port.

For the same reason, auxiliary propulsive power on the water is superfluous. For example, take the case of a flying boat of 12,500 lbs. displacement; the weight of a petrol engine reversing gear, stern tube and propeller of sufficient power for, say, eight knots, a probable minimum speed to allow for steerage way and adverse currents, would be of the order of 2 per cent. to 3 per cent. of the gross weight. Also a water rudder would be essential, which in combination with the water propeller would lead to a considerable increase in air drag.

Bilge Pumping Arrangements

A certain amount of water leakage takes place in all classes of ships. As would be expected, the flying boat hull, necessarily of a light form of construction and working under severe sea conditions, would prove no exception. The leakage at any time will depend upon the design, workmanship, the materials used in the construction, age, and the treatment it has been subjected to in the past.

In a well-designed hull in good condition the normal leakage is quite small. In the case of the "Fury," when riding at her moorings in a gale which lasted for three days, the leakage was barely a bucketful per day. Under normal conditions the leakage was so slight that it had to be mopped up with "waste."

The arrangements adopted to deal with the leakage depend upon the size of hull, number of bulkheads, whether of single or double bottom construction, and steps opened or closed. In each watertight compartment provision should be made for the water to drain to the lowest point relative to still-water level. To ensure this, the passages should be of ample area to prevent choking up, and be easy of access. Further, in double-bottom hulls, the inner and outer skins at the keel should be kept well apart to avoid the formation of a wedge-shaped pocket which cannot be easily drained. The pump, which may be either of the plunger or semi-rotary type, preferably the latter, should be placed amidships in a position get-at-able by any member of the crew, and as close to the bottom of the hull as can be conveniently operated, to minimise priming troubles.

In a single-bottom open step boat, the bilge suction pipe may be either a flexible rubber hose pipe of sufficient length to reach all the compartments, which are emptied in turn, or permanent duralumin pipes may be laid to each compartment. In both systems the suction end of pipe should be fitted with a shallow strainer, and to advantage a light non-return flap valve. The discharge pipe should be led overboard at a position on the ship's side well above the load water-line.

When there is a double bottom, which may extend from steam to after step and further be sub-divided into a large number of watertight compartments, any system will become more complicated. When this type of hull is stowed ashore, drain plugs on the outer bottom may suffice, but when lying for long periods at moorings, means must be provided for draining the space between the inner and outer bottoms. Otherwise, should leakage happen to be bad, useless weight would be carried in the flight.

Probably the arrangement of least weight would be to have fixed to the inner bottom stand pipes, taking care provision was made to allow the entry of air into the space between bottoms, to which could be attached the flexible rubber hose pipe mentioned above.

However, the simplicity or otherwise of any arrangement adopted will be influenced largely by considerations of weight and the duty for which the boat has been designed.

Power Plant

With the exception of the fuel supply to, and starting up of, the engines, only minor difficulties are experienced in the power installation which are not met with in the corresponding aeroplane.

In the more general case, where the engines are mounted between the planes, the main fuel tanks will be inside the hull and the usual gravity tank in the upper plane. On short range boats it may be possible to carry all the fuel either in tanks on the top plane or in the engine nacelles.

While this is a most desirable arrangement, giving increased cubic capacity in the hull for passengers and cargo and leading to a very simple petrol system, it should be remembered that inertia loads, due either to getting off or landing in a rough sea or with bad landings, are transmitted to the hull via wing root struts which are inclined at about 45 degrees and connect the foot of the engine mounting struts with the keel. There is therefore a strict limit to the amount of fuel which can be carried in this way if the weight of these and related members is not to become excessive. In some landing tests on flying boats carried out in America, the deceleration reached as high a value as 7 "g." With the aeroplane, on the other hand, this arrangement of fuel tanks might be advantageous, as the engines are generally vertically over the landing wheels.

In the general case, that is with tanks inside the hull, the suction head may be as much as 8ft. to 10ft. if the pumps are placed outside the hull in a position free from spray and other interference, such as from the wind screen in the forward cockpit. The delivery head may also range from 10ft. to 18ft.

Assuming the fuel is discharged to the gravity tank from the main tanks and thence by gravity feed to the carburettor, this may be done in three ways, viz., compressed air, centrifugal or plunger pumps.

Tanks under air pressure are excluded, as, apart from other disadvantages owing to the large cubic capacity required and the high delivery head, the weight would be excessive. If compressed air must be used, then a modified "Autovac" system should be adopted.

If centrifugal pumps are used placed outside the hull, then, owing to the large suction head, experience has shown they are unreliable, no matter how carefully all joints on the suction sides are made. For the same reason, priming is a difficult matter. To get over the suction head difficulty, the pumps may be placed inside the hull and level with bottom of tanks, and driven by a windmill through two sets of right angle bevel gearing. Due to the high pump speed, the gearing would probably have to be reduced in the ratio of about 3 : 1 to avoid whirling of the connecting shaft. Alternately, the pumps might be driven by electric motors, but this leads to further troubles. In any case, unless the pumps do not leak either when running or at rest, the petrol fumes due to leakage are a source of danger and discomfort to the crew.

On all counts, a well-designed plunger pump is by far the most satisfactory. It can cope easily with suction heads, without priming, up to at least 20ft. and any delivery head likely to be required. If a simple plunger pump is used driven by a windmill through a crank, and connecting rod, and valves automatic, these valves should be light discs, and given a small lift, and care should be

taken that there is no chance of any air lock on the delivery side inside of the pumps barrel or valve casing, otherwise, unless well primed, the pump will be most inefficient and may fail altogether to discharge.

The Swashplate plunger pump developed at the R.A.E. has an excellent performance, and is peculiarly suitable for use on flying boats. When the weight of fuel carried is of the order of 20 per cent. to 30 per cent., as it usually would be on long range boats, the overflow from the gravity tank should be selective, and under the control of the engineer. Thus by means of a simple indicator operated by the pilot, the change of trim with fuel consumption may be kept within small limits. For example, the "Fury" had tankage for 1,500 gallons of petrol. With tanks full and empty the C.G. travel was from .37 to .39 of the wing chord respectively, within which limits it could always be trimmed by the above arrangement.

When the flying boat services become established, the boats will either ride at moorings or lie alongside suitably designed wet docks. Hand starting gear under these conditions is awkward to operate and likely to cause considerable delay in getting under way. For these reasons it may be looked upon as a stand-by gear in case of emergency. Of all the various methods, mechanical or electrical, which may be proposed, the most promising and suitable to meet flying boat conditions is the small petrol engine starting set, in which the mixture is pumped into the cylinder at a pressure sufficiently high to turn the engine, the magneto on which also igniting the mixture. As weight is not of the same relative importance as in the small aeroplane, the compressor could be made of ample capacity to deal with any normal engine valve leakage. Such a set would not probably weigh more than 40 lbs. It could also be usefully employed in other ways, as driving a W/T generator or the bilge pump.

Operations

As commercial flying boat services, operating boats of suitable displacement and design, have not been established in this country, information as to the running and maintenance costs is not available; therefore a comparison in this respect between flying boat and aeroplane services will not be attempted.

However, a brief discussion of three practicable methods, in two of which a fair amount of experience has been obtained, and of the third, the most likely to be adopted in the future for large flying boats, will give a good indication of the relative costs from the differences in the organisation and equipment found to be necessary.

In the first system to be described, which was used almost exclusively during the late war, the boats when out of flight are stored in sheds in the usual way. Each boat has its own trolley, on which it may be wheeled from the shed, down the slipway to the water edge, and floated off, and *vice versa*. This system, while probably the most suitable and economical for boats of relatively small displacement, experience has shown to be unsatisfactory when the displacement ranges from five to fifteen tons, and to be out of the question for greater displacements.

The increase in initial cost and maintenance of sheds is almost directly proportional to the number of boats in the service, and to the increase in their dimensions, and as boats of fairly large displacement will be used commercially, the cost will be prohibitive. Further, costly slipways and trolleys will form a necessary part of the equipment, and a relatively larger and more skilled handling party is required than found necessary for aeroplanes. Even with careful handling the hull sustains more damage, due to the "floating" off on the trolley, than would take place under average sea conditions, thus unnecessarily reducing its useful life.

In this system the embarking and disembarking of passengers, handling of cargo and refueling is easy and safe.

In the second system the boats may lie at moorings in the most sheltered waters available, and only come ashore for repairs which cannot be conveniently carried out there, and for periodical overhauls. Under these conditions lesser shed accommodation and fewer trolleys would be required than in the first system, and would depend only on the number of boats in use, as one spare would be required for every three in flight. Provided the boats are properly moored, and engines, propellers and cockpits covered over to afford protection from spray, rain, etc., the system works fairly well. Boats not specially designed to meet these conditions have been moored out at the Scillies for a period of fourteen weeks, and were still in good flying condition. The fabric covering on the planes and control surface least of all will withstand such severe weathering and becomes very soggy. Before this system can become commercially successful, therefore, a great deal must be done toward increasing the durability without a serious increase in weight.

The main drawback to this system is the difficulty likely to be experienced in the embarking and disembarking of passengers and cargo. A motor-boat tender has to come alongside, and the transference under moderately rough sea conditions would in consequence be a somewhat hazardous undertaking. Fueling would be carried out from either a fuel boat or the motor-boat tender equipped as such connecting up by means of a flexible rubber pipe, and presents no difficulties.

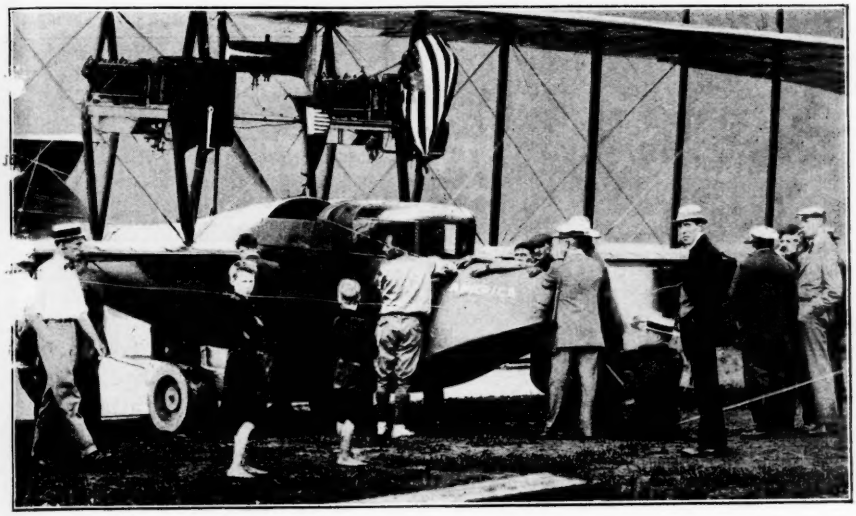
Before describing the third system it may be as well to indicate shortly the general lay-out of a large flying boat to meet the requirements. The experience gained from the "F" boats, and especially the "Fury," showed that so far as durability was concerned, the hull design was satisfactory, but the wing structure was much too fragile and complicated. As the result of the large area required with normal wing sections, further fabric covering must be dispensed with. The specification aimed at was as follows:—Three engines and propellers abreast mounted in the top plane, biplane wing structure, small gap-chord, deep section wing internally braced, span of upper plane considerably greater than that of lower plane, chord tapering from wing tip to centre section. External wing bracing from outboard engines to hull only. Owing to short span of lower plane large wing-tip floats to be fitted, wing tips not to extend beyond floats, wing covering either corrugated double skin wood or duralumin. In this way it is hoped to obtain a robust, compact and durable structure, with little sacrifice in performance.

Assuming boats as outlined above are available, and there are no insurmountable difficulties in the way, then probably the best arrangement would be to have a floating landing stage, with a gangway connecting to the shore; this in the first place would overcome all the tidal difficulties. The landing stage or floating dock, which as a mobile unit was submitted to the Air Ministry by the lecturer early in 1917, would be divided into at least two separate docks—one wet dock for outgoing or incoming boats, and the other, which by simple means would be used as a dry dock, for executing repairs, changing engines, etc. These docks would be probably at right angles to keep down the overall dimensions, and trim maintained by ballast tanks.

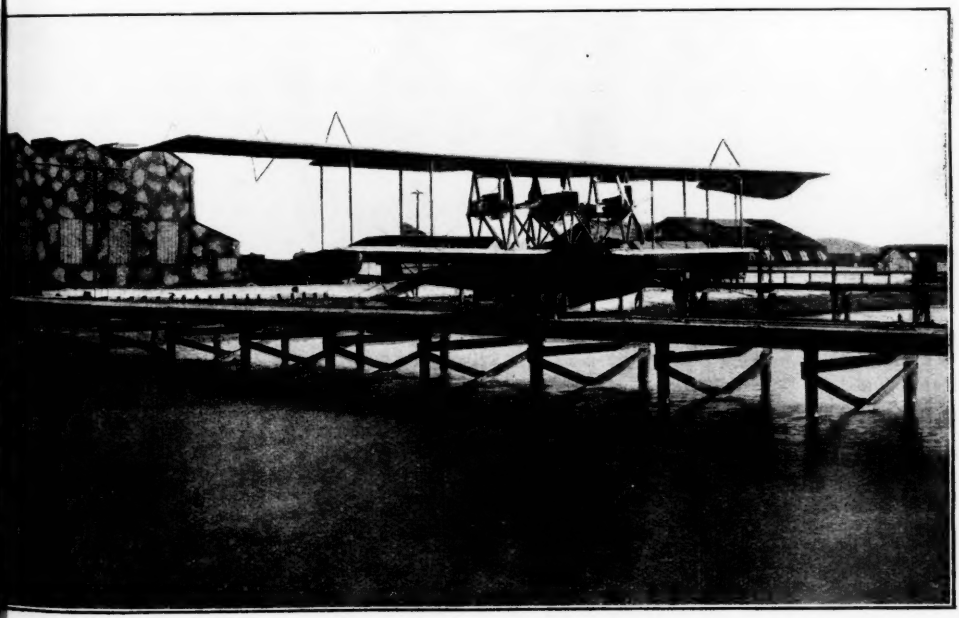
Normally, boats would lie at moorings, and only come into dock to take on board passenger cargo and fuel.

With this system shed accommodation, with the exception of shore workshops, would be unnecessary.

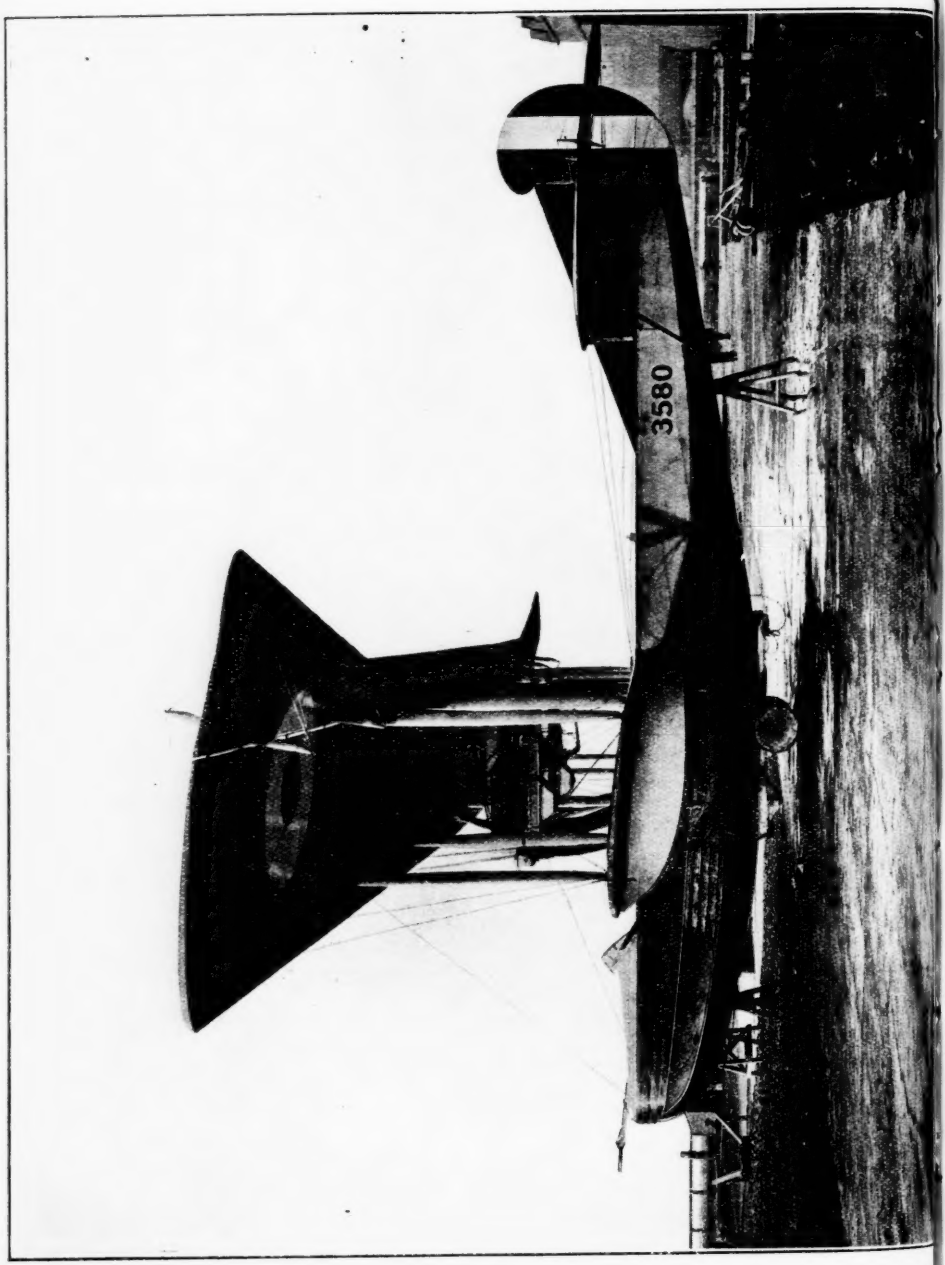
For those who have experience in the organisation and of running costs of aeroplane services the above brief discussion should suffice. The details are easily filled in. Bearing in mind that large flying boats will most likely be used in a commercial service, a rough estimate of the relative operating costs might be obtainable.



The famous "America" on which the late Col. J. C. Porte was going to attempt to fly the Atlantic in 1913. Colonel Porte facing the camera.



Porte "Baby" on Slipway.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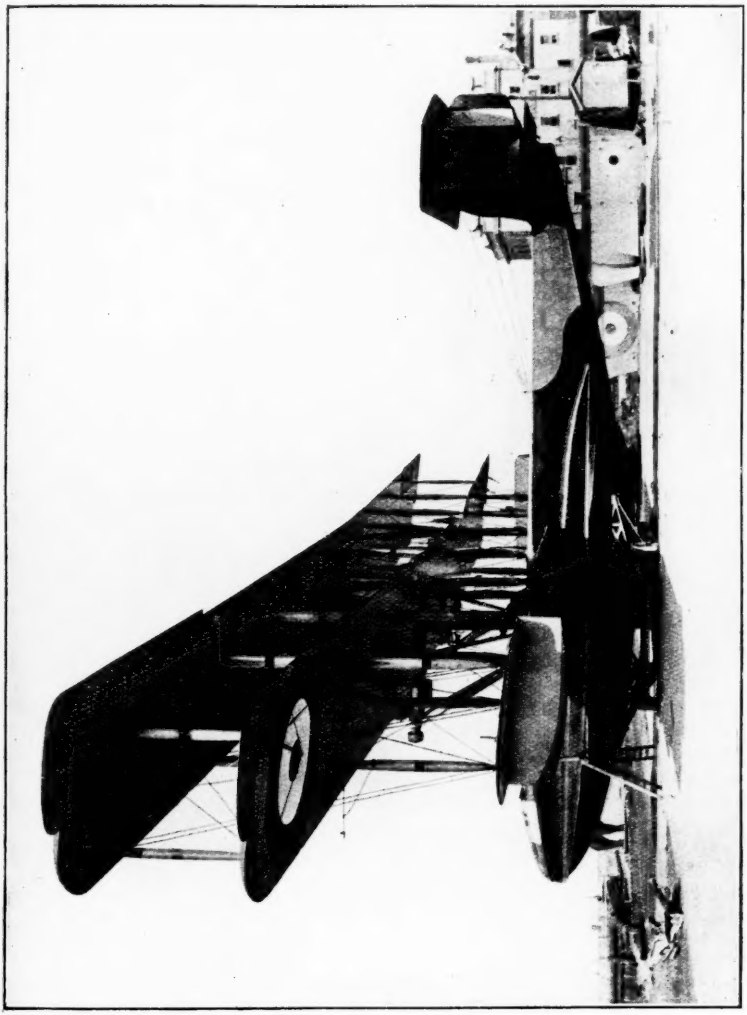

Felixstowe "Fury."

Discussion

Mr. G. S. Baker said he was very glad to see from the Paper that they could reckon Major Rennie as a disciple of tank methods. One came to that conclusion because Major Rennie had based a good many of his arguments on data which had been obtained from the tank. Dealing with the Author's introductory remarks as to the development of the flying boat being at a standstill, Mr. Baker said he did not think they would all agree with the reasons given in the Paper. He himself believed that the development of the flying boat would always be slow until there was a good technical man, a trained naval architect of some sort, holding a responsible position at the Air Ministry in charge of such work. Until that was done it was almost impossible for technical work to proceed properly; secondly, we must avoid concentrating practically all the experimental work on one type of boat, as was done in Porte's time. If we wanted to develop, we must give every type that showed a reasonable chance of success a full-scale experiment to demonstrate its feasibility. Since this had been started he believed it could be said that we had stopped building the " F " type boat, which had been replaced by a number of other types developed by individual builders. He believed he was right in saying that no " F " type boat had been built during the last 12 or 18 months, and possibly during the last two years. The Author, in dealing with hull lines and dimensions, had stated that tank tests were at variance with full-scale tests as regards the loading which it was possible to put upon the hulls. He believed that was the meaning of the sentence referred to, and he asked Major Rennie for the specific data on which he had based that statement. He himself was very keen to get as much comparison as possible between full-scale work and model work, and he knew of no definite data to support the Author's argument. Passing to the statement in the Paper with regard to the " Felixstowe Fury," in the same paragraph, it was desirable to put the facts on record so far as the tank was concerned in that matter. The Author had stated that " the modifications carried out on the new types of hulls evolved at Felixstowe were arrived at from full-scale experimenting, and with the exception of the ' Fury ' hull, tank tests on the corresponding models were not available at the time." The National Physical Laboratory, said Mr. Baker, had nothing whatever to do with the design of the " Fury " hull; it was designed by Colonel Porte, built by him, and sent to the tank in September, 1918, for tank tests after it had been built. The model was then tested at the tank, and it had been shown that its resistance could be lowered 25 per cent. over the whole of the speed range right up to 32 knots, and a letter had been received from Felixstowe stating that some of the modifications suggested by the N.P.L. were being embodied in the hull. The N.P.L. had reported, on the tests made, that the hull was liable to porpoise at 27 knots, and that no change which was feasible on the hull at that time could possibly eliminate that porpoising; the crash which occurred was due to the hull being rocked because of its facility for porpoising. The tests were made at a load which was not the proper load, but it was the load given to them, and the centre of gravity was not the centre of gravity. The impression received from that paragraph, that the N.P.L. were in any way responsible for the " Fury " lines, was not correct, in view of the facts he had given. Passing to the naval architecture side of the Paper, the hull weights and stresses, he had seen the " F " boats in the course of construction, and also what had come to be known as the " Linton-Hope " boats, and he was quite certain that any other naval architect who had seen them would say there was no comparison between them. The Linton-Hope work was really scientific compared with the work which was done on the other boats, and, further, there was no doubt at all about the superiority, from a weight point of view, of the Linton-Hope type of

construction. Also, he believed that Major Hope had had a great deal to do with the big improvement which took place in the construction of the later " F " types of boat. He himself knew that Major Hope had introduced very many changes, and had eliminated the cutting of many of the long strength members, and so on.

With regard to the elasticity referred to, he had noticed that the Author had referred to it as the " so-called " elasticity, and he was glad that he had, because he thought the term was applied a little wrongly to Linton-Hope's work. The real difference between the " F " type and the Linton-Hope type of construction was that in the " F " boat they built up stiff main transverse sections connected with a number of comparatively stiff fore and aft members. Between them were comparatively large areas of bottom more or less unsupported. In the Linton-Hope work practically all the bottom was supported by transverse members only two or three inches apart, which broke the bottom up into small partially-supported pieces all connected to elastic members. In the " F " boat hull, when running on the water, you could see the big patches of bottom panting in and out, and in one or two cases of damage he had seen the damage occurred because of that panting of the bottom between the stiff members. But the real difference between the two types, was not that one was elastic and the other not elastic, but in the manner in which these stresses were spread over and taken up by the main structure. There was another point which had not been touched upon by the lecturer, and it was a real defect on both the " P " and " F " type, namely, that one could not get at them inside. For the impact experiments on the " F " body he had been given a hull which was not quite three years old, and the inner skin was rotten in places, not because of bad work, but because it was almost impossible to get there to keep it clean. One could hardly get at some of the structure inside the boats, and if the hulls were to last more than two or three years they must be accessible inside. He knew of none which were really accessible.

Captain D. NICOLSON thanked Major Rennie for having brought the Paper on the subject of flying boats before the Society, as up to the present very few members had tackled the subject. He also thanked Mr. Baker for his remarks, and agreed with him that the reason why flying boats had not advanced much since the war was because the Air Ministry did not employ a competent naval architect to supervise the designing and construction of these boats. He would go further; and stated that the hulls should be built by motor boat and yacht builders, and not by people who had not had previous experience in hull construction.

In answer to one of Mr. Baker's queries, as far as he was aware the Air Ministry had not ordered any of the " F " boats since 1918. Major Rennie had said that a big number of flying boats were a failure. He personally only happened to know of one, and he would like to know the others which turned out to be complete failures.

Major Rennie had also said that the laying off the lines was by no means easy. He entirely disagreed on that point, as the operation presented no difficulty to a naval architect. In fact, he stated that any boy completing his apprenticeship had to draw similar lines for boats and yachts; and as the process was so elementary, but proved difficult to the non-professional man, it was evident that only naval architects should be asked to lay out such lines.

Major Rennie had said that the extraordinary behaviour of the hull of the Porte Super-Baby in rough seas was due to the buoyancy and the lines adopted. Captain Nicolson said in his opinion the obvious increase in size would increase the seaworthiness. He believed that all that part of Major Rennie's description of the trimming moment came down to the fact that if the boat was properly designed it would not nose dive.

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He also stated that the steps of the F.5 could have been altered to make this boat stable in the water; he admitted that this was advisable, and had stated that it could easily have been done, and he (Captain Nicolson) would like to ask why it was not done?

With regard to the question of the timbers in the Porte or fuselage type being continuous from chine to chine; this was not the case in the majority of the "F" boats, but was stopped at the keel, thus causing leaky boats, and continual trouble.

Captain Nicolson had to introduce intermediate timbers in the early "F" boats, to prevent the planking from coming away from the keel.

On the subject of bulkheads, Major Rennie had said that he thought these would give way with the inrush of water. He (Captain Nicolson) considered that bulkheads could easily be fitted that would withstand the normal inrush of water. This subject he dealt with fully before the Society, and made many suggestions, among which he proposed a bulkhead committee as set up by the shipbuilders in this and other countries.

Major Rennie had also said that double bottoms should only be fitted in way of the passenger accommodation. He (Captain Nicolson) did not agree, because in ordinary naval architecture they always ran double bottoms the whole way of the water line; and he thought the same should apply to flying boats.

The reason the lecturer had given for the supposed superiority of the Porte type over the Linton-Hope type, was because the boats rested on the keel. He had never yet seen one of these boats resting on the keel when in a trolley; thus causing the planking to spring, with the result that the boats leaked when put in the water. He had been to Felixstowe, and had found that each boat had a particular trolley to fit that particular boat, and even with this precaution the boats did not rest on the keel; this could easily be seen by examining the plans. After his visit to Felixstowe he designed a new false keel to keep the planking up to the original keel; this was bedded in red lead to make a water-tight job.

Another point which the speaker considered showed very bad design in the original F.2A. and F.3, was that all the sides aft were made of fabric; to a naval architect that was not a good job, and all the latest F.5's had diagonal planking, which was of his own design.

With regard to the number of metal fittings used in the construction of the Porte type, in his opinion they were far too numerous, which was from the production point of view very bad; and this might be one of the reasons why the "F" boats are not in the list of boats being built at the present moment.

It was said that the fittings did not rust away in a short time, but he had his doubts about that. If they did rust, it would probably mean pulling down half the boat to get some of the fittings out.

With reference to the streamline form, he had no doubt that the Linton-Hope type was much the superior form.

Regarding the boat which Major Rennie had pointed out was built on the yacht or motor boat form of construction, which he did not like, he (Captain Nicolson) would like to inform the Author that that boat was not built on the yacht or motor boat principle; showing that Major Rennie was not acquainted with motor boat construction.

As to the Author's reference to the use of three-ply for small bulkheads, the speaker did not agree; he did not think that three-ply was suitable for building bulkheads.

In conclusion, he said that although his remarks had not perhaps been such as Major Rennie would have liked to have heard, he hoped they would not be considered to be directed to Major Rennie personally; he was speaking from the point of view of the naval architect.

Mr. MANNING joined with Captain Nicolson in his request for particulars of the boats which were said to be unsuccessful. He believed the Author would find that several quite successful machines had been produced in the last two years by private firms. With regard to the table of percentage weights published in the Paper, he would like it to be realised that the figures attributed to the F.5 were not representative of the standard F.5 as produced in large quantities towards the end of the war. The figures referred to a special F.5 built at Felixstowe, and, so far as he knew, only one of this type was ever built. This special F.5 necessarily had to be considerably strengthened and otherwise altered to make it fit for general service, and it was the modified type which was turned out in large quantities. Comparing this machine with one of the " Linton-Hope " type of approximately similar size, P.5, the structure weight of the standard F.5 was 47 per cent., and of the P.5 35.8 per cent.; the useful load percentage of the standard F.5 was 30 per cent., and of the P.5 39.3 per cent.; the weight, empty, of the standard F.5 was 9,100lbs., and of the P.5 7,439lbs., and the useful loads were 3,900lbs. and 4,750 respectively. The speed of the " Linton-Hope " type of machine was about 13 or 14 knots higher than that of the F.5; the speed of the standard F.5 with full load at sea level being 76.5 knots, and that of the P.5 under the same conditions 90 knots, in both cases with Rolls-Royce Eagle engines. The standard F.5 was a very useful flying boat, and did very good work towards the end of the war, but it was clear that its performance as an aeroplane was inferior. He drew special attention to the figures, as the great superiority of the P.5 was largely due to its having a " Linton-Hope " hull. Also the tail area of the P.5 was 196 sq. ft., and not 143 sq. ft., as had been stated. The tail volume coefficient was, therefore, .42 and not .31. The lecturer had given a description of the complicated elevator movements required to get an " F " boat off the water. It was one of the advantages of the " Linton-Hope " type, as developed by Mr. Baker, in the tank at the N.P.L., that none of these movements were necessary. Such a machine could be started from rest on the water, and could be flown off without the elevator being touched. Major Rennie, in his description of the " Linton-Hope " type of construction, had stated that these hulls had three-ply formers in the bottom. He (Mr. Manning) pointed out that the use of three-ply in such positions had been obsolete for several years, and that recent hulls had no three-ply at all in their construction. The lecturer had made a great point of the wing root structure on the " F " boats. It might be necessary in this type, but all the " Linton-Hope " type boats got on perfectly well without it, and this type of boat could be put on a trolley in just the same manner as the " F " type; in fact, they were always handled in this way. The risk of damage was about the same in either type. On the other hand, the wing root structure was very heavy, and obstructed the interior of the hull. The lecturer had also suggested that in hulls of the " Linton-Hope " type the tail setting did not remain constant, owing to variation of temperature, etc. He (Mr. Manning) could assure the lecturer that he was mistaken, and trouble of this type did not occur in the " Linton-Hope " hull. Again, in his description of the " Linton-Hope " type hull, the lecturer had referred to three-ply formers, and to the pocket between the inner and outer skins. As a matter of fact, both these defects had not been " more or less " eliminated, but " completely " eliminated in more recently designed hulls. As to hull weights, the Author's F.5 weight referred to the special F.5 built at Felixstowe, and not to the standard F.5 as used. He did not know the exact weight of the latter hull, but it was approximately 2,500lbs., or 1,000lbs. more than the other. The lecturer had also stated that

the somewhat smaller size of the " Linton-Hope " type would make it less seaworthy than the " F " type. There was no doubt that the " Linton-Hope " type of hull as fitted to the P.5 was considerably more seaworthy than the " F " type hull of similar displacement; this had been proved. It was clear that Major Linton-Hope, when he designed the hull, knew quite well what he was doing, and did not put the load water line too high, as suggested. The speaker had noticed that Major Rennie had recommended three-ply for use on bulkheads of wing-tip floats. This material was quite unsatisfactory for this purpose, and was not used on any modern design. He agreed with Mr. Baker when he had said that the accident to the " Fury " would not have occurred if that machine had been fitted with a hull of the " Linton-Hope " type. The lecturer's curve showing hull weights was interesting, but it should be remembered that, however suitable these hulls were for the special conditions at Felixstowe, they required considerable strengthening if used for general service, which would increase the weight considerably. The fact that the " F " type of hull was unsuitable for the fitting of a double bottom was an exceedingly serious defect. The lecturer had attempted to suggest that a double bottom was not necessary, but he (Mr. Manning) did not think that anybody who had had experience with flying boats would agree with this. A boat which, fully equipped, might be worth, say, £50,000, should not be sent to sea in a condition which would lead to total loss if it landed on a piece of sunken wreck. He thanked the Author for his Paper. The description of the early experiences at Felixstowe was exceedingly interesting, and the collection of experiences of this sort would be very valuable in connection with future design.

Captain G. T. R. Hill, referring to the point raised as to the length of the bodies of seaplanes as compared with those of land-going machines, said it would be noticed that, on the whole, the length of the bodies of the seaplanes was shorter than that of the land machines. The tail volumes were, on the whole, about the same, so it must mean that the seaplanes had larger tails, and thus heavier elevator control. The fact that they had shorter bodies seemed to lead to the serious disadvantage of greater wing span; he could see from the tables that this was so, and that the aspect ratios of the seaplanes were in excess of those of the land machines. The bodies were apparently built short in order to keep down the weight, so that the whole design was inferior to that of the land machines in that the serious disadvantages of the extra span had to be put up with in order to keep the weights down to the same sort of figure as on the land machines.

Again, the wing weights apparently were slightly less on the seaplanes than on the land machines. From the higher aspect ratio it might be inferred that lower factors of safety were worked to on seaplanes, and he would like the Author's opinion as to whether that was so or not.

With regard to the trouble of getting the machine trimmed fore and aft, he had often wondered why, in these experimental machines, the designers did not allow for a small sweep backwards or forwards of the wings, to be obtained by packing out the front or rear spars at the roots, thereby altering the centre of gravity position so that the required amount of stability fore and aft might be obtained on trial. The same applied to land machines.

He would also like to know whether there was any possibility of designing seaplanes which could enter the water at a comparatively steep angle; *i.e.*, whether a hull could be designed which, while having the necessary qualities of seaworthiness, would allow the water to absorb the energy of the machine falling with a speed whose vertical component was 10, 15 or even 20ft. per second. It seemed that seaplanes would be much safer if they could come down at a steep angle when alighting on a small area of water in emergency. Did the Author

think it possible to allow a machine to sink, say, 4 or 5ft. after first touching the water, in order that the vertical velocity might be destroyed by the pressure of water on the bottom.

With regard to controls, there seemed to be still a certain amount of discussion as to the exact cause of the accident to the "Felixstowe Fury." He believed that accident, and accidents to other large machines, had been due really to the lack of proper control. On large machines nowadays the control surfaces were first designed of such size that the machine would have reasonable control. Then it was found that the pilot was not strong enough to operate the surfaces of that size, and they were cut down until he could do so, which necessarily resulted in machines having inefficient control. The result of this lack of knowledge as to how to provide adequate control on large aeroplanes might be likened to feeding a man on raw potatoes because one did not know how to make the kitchen range work; it was not only very unpleasant, but also highly dangerous.

Major BUCHANAN considered it only fair to say that the "F" type boats were used by the Admiralty throughout the war, and had performed excellent service for the Admiralty and the R.A.F. He did not wish to diminish what had been done by the late Major Linton-Hope, with whom he had been in close contact at the Air Ministry and at the Admiralty. While he agreed generally with the naval architects on the question of the design of the "F" type boats, at the same time, he thought it was going rather a long way to claim that the present type of so-called flexible hull was a complete solution. His reason for saying that was that, whilst we had a large experience of the "F" type boat, our experience of the "Linton-Hope" type of hull was very limited indeed, and therefore he felt it would be rash to say that the latter constituted a complete replacement of the "F" type of machine. With regard to useful loads, the Author had given the average useful loads taken by the aeroplane and the seaplane. He believed the Author had taken the average of certain things which were not comparable, and in conclusion he had said that the average useful load of the flying boat was 40.5 per cent., against only 38.5 per cent. for the aeroplane. In the first place, the figures were not quite correct, and he felt that the conclusion that the flying boat carried the same useful load as the aeroplane, was scarcely justified. In fact, he considered that a very strong case could be made out the other way. He could not agree with the Author that it appeared logical that the most suitable type of aircraft to develop in large sizes was the flying boat. One of the reasons was that already stated, namely, the useful load carried. He pointed out that for the flying boat greater engine weight had to be provided. He was also not sure that he agreed with the lecturer in his condemnation of the single-engine flying boat. It had certain definite and useful functions, and he did not think it was fair to say that the single-engine flying boat was not satisfactory. For the purposes for which it was used, and could be used, he believed that on the whole it was doing reasonably good work. In conclusion, he thanked Major Rennie for his very interesting paper.

Mr. W. A. WRIGHT (British Electric Co.) raised the problem of the cost of construction of these boats. Major Rennie had shown that in the latest type of boat developed by Colonel Porte the planking and timbers were carried round in a rather complicated form. He was inclined to think that the cost of construction in that case must have been very high, and that was a point which would seriously affect the development of flying boats in the future. In the old "F" type of boats there was a large number of fittings, which would increase cost unless they could be produced in very large quantities, but even if they could be produced in large quantities at a reduced price he was inclined to think that the "Linton-Hope" type of hull could be produced, size for size, cheaper than the "F" type. Major Buchanan had also mentioned that the principal duty of a

flying boat was to fly. He believed they had always agreed with that. He himself was always trying to reduce the weight of the hulls, but one could not overlook the fact that large flying boats had to be efficient boats. The difficulty of housing machines when not in use was very great, and the type of machine that had been considered in the Paper was not of much use unless it could remain afloat for weeks on end when not required for flying. That led him to say that, on the question of whether these boats should be designed by aircraft designers or naval architects, after all said and done, the most useful thing was to have co-operation between the two. So far as he could see, that had resulted in by far the best results obtained up to the present. He thanked Major Rennie for his Paper.

Mr. E. GIBSON KNIGHT (*communicated*): From the manufacturing point of view, one of the most essential differences between the Felixstowe type of construction and the Linton-Hope construction is the fact that while the former type employs a large number of metal fittings, the Linton-Hope type, with the exception of towing plates and strut attachments, requires no metal fittings in its construction. The metal fittings in the F.5 flying boats cost about £200 per machine to produce when manufactured in quantities of some hundred sets, which enabled almost all parts to be stamped out by means of dies, and I think it is no exaggeration to say that to produce a similar set of fittings to-day, in the quantities of a few sets at a time, would cost not less than £400 per set. The cost of the boat-building work on the two types is about equal and it therefore appears that the Felixstowe type will—size for size—always cost more than the Linton-Hope type by the cost of the metal fittings and, judging from experience on the F.5, this would mean an addition to the cost of about 25 per cent. unless they can be built in very large quantities.

The later type of Felixstowe construction in which the planking joints at the chines and fin tops were eliminated by bending the timbers and planking round these points would be even more expensive, as a very elaborate bending jig would be necessary for the bending of all the timbers, every one of which requires to be slightly different, and every timber must be steamed. In addition, while it is understood that the outer planking was put on diagonally, thus enabling it to be bent on the actual hull itself, this necessitates the steaming of every plank as it is fitted into place. One hesitates to give any idea as to the extra cost of this method of construction, but it must be absolutely prohibitive when compared with the Linton-Hope construction, in which no steaming is required on either the timbers or the planking, with the possible exception of a few timbers right at the tail end of the boat. Further, the sole object of this later type of Felixstowe construction was to eliminate the planking joints at the fin chines and fin tops owing to the large amount of trouble experienced with the Felixstowe types through unsatisfactory joints at these points. In the Linton-Hope type of construction this trouble is not experienced and there is, therefore, no object in adopting a costly and elaborate system of obviating these joints.

Major Buchanan mentions that the principal duty of a flying boat is to fly, and in this I absolutely agree with him, and think there is some tendency on the part of the Air Ministry to insist on too great a perfection in the hull construction at the expense of weight and consequent deterioration of air performance. On the other hand, one has to realise that the difficulty of housing large flying boats is very great and they should undoubtedly be capable of being moored out for some weeks when not required for flying.

While one cannot agree with a very large number of the Lecturer's opinions, one certainly agrees that it would be unwise for the Air Ministry to devote themselves entirely to one type of construction to the exclusion of the others. It must be pointed out, however, that while the "F" boats have already been produced in enormous quantities and have had several hundreds of modifications

and improvements effected in them, the resultant machine was even then inferior to the first machine of this size fitted with a Linton-Hope hull, and on which practically no modifications were carried out whatsoever, and there is little doubt that when the improved type of machines now under construction are tested, the superiority of the Linton-Hope over the Felixstowe construction will be even more marked. In conclusion, one would thank Major Rennie for a most interesting, if somewhat controversial, paper.

The CHAIRMAN, dealing with aerodynamic structure, quoted the Author's remark that "It has been said frequently of the 'F' boats that the use of stabilisers on the top plane is a very inefficient aid to lateral stability. They were never really intended as such, as it was well-known an increase of dihedral angle would be much more effective." He had never understood what stabilisers were for, and if Major Rennie could give him some idea as to why they were put on it might help in the future.

Major RENNIE intimated that he would prefer to reply to the criticisms raised in writing.

A hearty vote of thanks was accorded the Author for his "very interesting and apparently very controversial Paper," and the proceedings terminated.

MAJOR RENNIE'S REPLY

In reply to Professor Bairstow, with regard to the effect of vertical fins on the top plane of flying boats, I am afraid I haven't the courage to explain the aerodynamic effect of a vertical fin above the C.G. in the case of, say, sideslipping to such a well-known authority on aerodynamics. These fins originated with the early Curtis boats, and have been retained for reasons given in my paper.

In reply to Mr. Baker, he says they could reckon me as a disciple of tank methods, because a good deal of my arguments were based on data obtained from the tank. With the exception of the curves showing the water performance of a hull, my arguments are based entirely on the results of full-scale experimenting. I think I have made my position in this respect quite clear in the opening paragraph under "Hull Lines and Dimensions."

He says that in Porte's time experimental work was confined entirely to his type of boat. This is not the case, as one firm at least, The Norman Thompson Flight Co., were concentrating on flying boats at the same time. They had little success, as their hulls were of the old Curtis type, which had many serious defects.

Again, he says that for the last 12 or 18 months, no "F" type boats had been built, as they had been replaced by other types developed by individual builders. I am afraid Mr. Baker is unacquainted with the true position. During the war about 500 "F" boats were ordered from various firms. Of these, say, 200 were delivered and used up to the Armistice, after which a certain number were cancelled and the remainder delivered to stores. Some of the latter are still being reconditioned and delivered to the Service. There is no R.A.F. squadron equipped with any other type of flying boat, and no other type has, so far, got further than the experimental stage.

It may be of interest to state that the U.S. Naval Air Service are using F.5's exclusively, and have found them to be most satisfactory.

With regard to hull loading, I think it is quite clear what I mean. I have no specific data. The conclusions are based on the experience gained with a large number of boats, under all sorts of conditions. All data cannot be expressed in mathematical symbols, as Mr. Baker may know. If Mr. Baker will read more carefully the part dealing with the Felixstowe "Fury" I think he will realise there was no intention of giving the tank credit for the hull lines. While quite realising the position of the steps on the "Fury" could have been altered to help towards the elimination of porpoising, the lines generally were found to be most satisfactory, but could be further improved as explained in my paper.

I think I have acknowledged the assistance obtained from the tank in suggesting modifications to reduce the water resistance.

With regard to the crash of the "Fury," it is quite obvious many important facts are not generally known. Prior to the accident she had done about 30 hours flying, and in every way was found most satisfactory. Under the control of expert boat pilots, such as Colonel Porte, Majors Hallam, Hobbs, Wright, and Cooper, porpoising to any serious extent was absent when taking off. The tendency was there, but sufficient elevator control was available to keep it down. Owing to the tail plane being well covered by the propeller slipstream, there is more elevator control, over the range of speed between the hump and taking off, than probably is generally realised.

The C.G. position had been determined by weighing in the usual way, contrary to what is stated in the report on the accident.

Several months before the accident Colonel Porte and myself were demobilised, there was therefore no technical officer in charge. No one knew where the C.G. ought to be or took the trouble to find out. The result was that the boat was loaded up with spares, etc., and the fuel, of which there was tankage for 1,500 gallons, most likely distributed in the tanks such that the final C.G. position was at least at .5 of the chord. Further, from a reliable and intelligent member of the crew, the late Major Moon attempted to take her off, as he had done on a previous occasion, before the minimum safe flying speed had been reached. As loaded, she was underpowered, there was therefore little available h.p. for acceleration, once clear of the water, with the inevitable result. While possibly this might not have occurred had the arrangement of steps given stability on the water and the water moment such that the air control was insufficient to allow of the boat becoming air borne in a stalled condition, it may be mentioned that a stable hull is only such under the conditions for which it has been designed. While small variations in loading and C.G. position are of little importance, sufficient deviation from those conditions may easily arise in actual practice, such that an otherwise stable hull may tend to porpoise. Also, for example, in a long heavy swell it is quite possible for any boat, no matter what step arrangement is adopted, to attain an angle of incidence corresponding to the stalled condition.

While Mr. Baker has done a great deal towards the elimination of porpoising, the problem is by no means completely solved.

Before replying further to Mr. Baker, it may be as well to say a few words with regard to the part in the design and construction of a flying boat which could come within the province of the naval architect. From Mr. Baker's and Captain Nicolson's remarks one may rightly infer that unless one is a naval architect one can never hope to design a successful flying boat hull. With this I am in disagreement, as is also Major Buchanan. In the first place, it should be understood clearly that a naval architect in the true sense does not include a ship's draughtsman or small boat builder. I take it Mr. Baker is hinting at the latter when he says the Air Ministry should employ a competent naval architect.

The knowledge of naval architecture required in the design of a flying boat hull is of a quite elementary nature, and from the purely theoretical side is well within the capabilities of the average aircraft designer. Apart from a certain amount of the detail design, the hull design cannot be separated from the aerodynamic or structural design of the complete boat, and up to the present this knowledge has not been possessed by those dealing with the hull design alone.

Mr. Baker says, "There was no doubt at all about the superiority of the Linton-Hope type of hull from the weight point of view." I am afraid a mere statement like this is not very convincing. The weights given of the F.5 were actual and not estimated, and the hull of this particular F.5 was, with the exception of that of the "Fury," the best hull designed and built at Felixstowe. This is not just

my own opinion. Experience showed it to be such. Major Linton-Hope had nothing whatever to do with the improvements incorporated in the design of this hull. The Porte I., the prototype of all the " F " boats, was under way before he joined the Air Dept., Admiralty.

With regard to Mr. Baker's contribution to my paper, setting out more fully the differences in the construction of the " F " and " P " type boats. The fact remains that the Porte hulls stood up and are still standing up to the work for which they were designed, in spite of all the adverse criticism and condemnation which they have been and are subjected to. Admittedly there are many faults and weaknesses in the Porte hulls, but there are just about as many in the Linton-Hope hulls. The naval architect is no more infallible than the engineer, as, for example, witness the accident to the rigid airship R.38. Again, the Porte hulls were built on a production basis, and given out to firms, in many cases, with no experience of boat building. Also a lot of unskilled labour was employed. Whereas the P.5 hulls were built by a " pukka " firm in this class of work.

The difficulty of access to the hull bottom inside the hull has always been realised, and when a large quantity of fuel is carried, seems inevitable. As mentioned in the paper, the fuel may be carried in tanks on the top plane, thus clearing the hull. The same trouble is likely to be experienced when hulls of moderate displacement are fitted with double bottoms.

In reply to Captain Nicolson, the chief reason for the stoppage of " F " boat construction is given in my reply to Mr. Baker. Further to which I might add that few, if any, Air Ministry officials were aware of the improvements incorporated in the design of the " Fury " hull. As these improvements led to a hull greatly superior to that of the " F " boat hulls, it is most unfortunate that the development of this type of hull has not been proceeded with, more especially when one realises the achievements and performance of these boats.

With regard to the failures of other hulls, Captain Nicolson will realise I am unable to make public their names, but I will be pleased to give him privately the names of at least six.

I disagree most emphatically with Captain Nicolson when he says, as he did at the discussion, that the setting out of the lines of a hull could be done by the office boy. I should have thought that a man who has been more or less connected with ship work would have realised that the setting out of the lines is divided into two parts. Firstly, knowing what to set down, and secondly the mechanical process of drawing them on paper.

If the former can be done by the office boy, then evidently the tank is unnecessary and experience of no account. In other words, we are all wasting time and money chasing the obvious. The latter is comparatively simple, requiring only an elementary knowledge of geometry and accurate draughtsmanship. On the other hand, such work, in my experience, is given to an experienced draughtsman, as it will be realised that a fair amount of work is detailed in the D.O. before the lines are laid down, faired and checked in the mould loft.

Again, Captain Nicolson said he didn't know of any ship in which the centre of buoyancy wasn't vertically under the C.G. With this I agree, but the trim in still water may be very different to that required. The desired L.W.L. is then obtained by, usually, a combination of a small shift of the C.G. and a slight alteration to the under-water lines.

The increased seaworthiness of the " Fury " hull over the smaller " F " boats was obviously partly due to the increase in dimensions. However, the marked improvement in this respect was due to the reasons given, which are based on experience and not on opinions.

I regret I am unable to reply to Captain Nicolson's comment on what I have

said about trimming moments, as I do not know to what he refers. I am afraid there is more in this world than meets his philosophy. The position of the steps on the " F " boats was obtained by Colonel Porte as the result of a great deal of experience taking off and landing flying boats under very varied weather and sea conditions. He probably did not realise that a more stable arrangement was possible. Just before the Armistice work was put in hand to fit steps on the F.5 in accordance with the arrangement developed at the tank, but the work had to be stopped owing to demobilisation troubles. It must also be remembered that the experimental F.5 had been flown before the design of the P.5 had begun, and before there was any full-scale test of the tank arrangement of steps.

It is obvious bulkheads could be fitted which would withstand a 12in. shell. It is entirely a question of weight. For an acceptable weight I very much doubt if bulkheads can be made to stand up to a serious crash in boats of the displacements in present use.

As flying boats become of relatively greater displacements, the fitting of a double bottom will be simpler, as the design will lend itself more readily to its adoption without a serious increase in weight. In any case, we are dealing with flying boats and not ships, the latter are not called upon to fly; the conditions are therefore not similar. The " F " boats were so designed that the hull, when on the trolley, rested on the keel, and was steadied at the bottom longerons. At the experimental station this always was the case, as the trolleys were fitted to the hull to ensure this. If such was not the case in the war flights at Felixstowe and elsewhere, then the fault lay with the trolley and not with the boat.

Captain Nicolson complains of the bad design in having the sides aft covered with fabric. Does he seriously believe Colonel Porte and his staff were unaware of this? As I have said, these boats were designed originally to operate from Harwich Harbour. They were used for submarine patrol and ship escorting. The maximum possible military load had to be carried, and the air endurance was continually being increased. Many sacrifices and risks had to be made to help in this direction, as the one and only object in view was to beat the Hun. This particular compromise was justified, as to give one example, the saving in weight over planked sides would add about half an hour to the air endurance at cruising speed. As may be seen, it wasn't a question of naval architecture.

Mr. Manning challenges the percentage weights of the F.5, and says the hull had to be considerably strengthened to make it fit for general service. As I was chief technical officer at Felixstowe, and therefore in a position to know accurately the history of this boat, I must contradict him. As I have already pointed out, this boat had been used extensively, and was well-known to be the best hull turned out at Felixstowe.

The F.5 was never put into production, which was a great blunder on the part of the Production Dept., Ministry of Munitions. Instead, the F.3 wing structure, the weight considerably increased to facilitate production, and adapters fitted to take either streamline wires or stranded cables, also permanent slinging gear incorporated, was fitted to a mongrel hull, a cross between the F.5 and F.3, and the resulting boat called the F.5. This was done solely because the F.3 was already in production (it never should have been), and the Ministry of Munitions were against a further change as jigs, templets, etc., were already made for the F.3. Therefore Mr. Manning's figures for weights of the F.5 are not accepted for two reasons; firstly, they do not represent the F.5, and secondly, the hull weight includes bulkheads, seats, etc., and consequently is not comparable with the figures given in my paper.

With regard to air performance, the following figures extracted from test station reports are of some interest. As the only complete tests of the P.5 available are those light and overload, these will be compared with the F.5 in the same condition.

LIGHT TRIALS.

	P.5. Two Rolls Royce Eagle VIII. Engines, 352 H.P. each.	F.5 (FELIXSTOWE). Two Rolls Royce Eagle VIII. Engines, 345 H.P. each.
Weight, empty	7,437lbs.	8,023lbs.
Total weight	9,210lbs.	9,630lbs.
Weight/sq. ft.	7.12	6.83
Speed at 2,000ft.	91 knots	88 knots
Rate of climb at 2,000ft. ...	672ft./min.	701ft./min.
Service ceiling	15,100ft.	17,400ft.

OVERLOAD TRIALS.

Total weight	12,511lbs.	13,306lbs.
Speed at 1,000ft.	80 knots	87½ knots
Rate of climb at 1,000ft. ...	353ft./min.	352ft./min.
Service ceiling	6,400ft.	9,200ft.

Without analysing these trials to obtain a better aerodynamic comparison, I think it is fairly obvious that the P.5 performance is only very slightly superior to that of the F.5. Further, the P.5 failed to take off with a load of 12,800lbs., showing that the F.5 may be overloaded to a much greater extent, which is not without its advantages. It should be mentioned that the wing section on the Felixstowe F.5 was different to that of the production F.5, which was R.A.E.14.

I have to thank Mr. Manning for correcting my figures for the P.5 tailplane. With regard to "taking off," while I am quite well aware of the fact that the P.5 has been taken off "hands off," this is not the usual practice, for reasons I have outlined elsewhere.

As the object of my paper was to set out some of the work carried out at Felixstowe, which came to an end early in 1919, when discussing hull construction, it would seem fair to compare the Porte type of hull with other types actually built prior to that date. At the same time, I took care to indicate improvements which subsequently have been found necessary on these types.

Three-ply formers between the planing bottom and shell were used on the N.4, which boat so far has not yet been tested. With regard to the pocket between the inner and outer skins on the P.5, I said this defect had been more or less eliminated. This is quite true, the many improvements carried out on these hulls have not yet been tested, and are either in an incomplete state in the shops or still on the drawing board. It is unwise to argue on hopes, no matter how promising they may be. On the question of tail sagging on boat-built hulls, I think if Mr. Manning will read carefully my paper, he will find that I never mentioned the Linton-Hope type of hulls, but referred to the earlier boat-built hulls, namely those of Curtis. Quite possibly the L.-H. hulls may exhibit this weakness when subjected to Service use.

Taking now the wing root structure, sufficient experience has not yet been obtained to say definitely that this structure is unnecessary in the L.-H. hulls. Experience has been gained with over 200 "F" boats, operating under Service conditions varying from stations in the North of Scotland to the Mediterranean. In all two P.5 boats have been built and used and have not been out of the hands of either the test station or Development Flight.

The curve of hull weights as shown provides hulls of ample strength to meet all reasonable requirements. This is not a question of my own opinion, but it is supported by the results of experience with a large number of hulls, extending over a period of several years.

I agree that double bottoms extending the whole length of the L.W.L. are desirable, but it has not been proved that with boats of the present displacements

they are essential and the increase of weights warranted. With boats much larger than so far built, as Mr. Manning suggests, at a cost of £50,000, I agree double bottoms should be incorporated, for reasons already given.

To say that "the fact that the 'F' type of hull was unsuitable for the fitting of a double bottom was an exceedingly serious defect" is a statement without the slightest justification. As I said in my paper, the "Porte" type of hull did not lend itself readily to the fitting of a double bottom. On the other hand, experience with these boats has shown that damage due to flotsam when taking off or landing is a most infrequent occurrence. I do not know of any flying boat yet flown which has a double bottom along the L.W.L. The Linton-Hope hulls certainly have not. In these hulls, the planing surface from the stem to the main step is additional to the hull proper, the skins of which wash into the hull about half way across the form; thus there is less than 50 per cent. of the area of planing bottom when there is a double bottom in which the two skins are separated to any extent, the remainder having only more skin thickness. At the step the two skins are separated from chine to chine as in the Porte hulls. In any case, blocks of timber are the most dangerous from this point of view. Empty beer bottles do not float, as Mr. Manning may easily verify, and I doubt very much if passengers on excursion steamers are allowed to take empty beer bottles on deck to throw overboard for amusement. Further, on this same point, Mr. Manning says "that anybody who had had experience with flying boats wouldn't agree that double bottoms are unnecessary." I think I am right in claiming such experience, as with the exception of about 1¼ years on full-scale research, I have been employed entirely on flying boat work since early in 1915. As a matter of interest I shall be very glad to compare my flying hours, time stationed on air stations, and sea experience generally with Mr. Manning and Captain Nicolson.

I think I am right in stating that Mr. Manning's flying boat experience has been confined to the P.5, of which so far two have been built. It is therefore of some interest to trace briefly the history of the P.5.

Colonel Porte had shown, by the end of 1916, that it was possible to design, build and successfully fly a twin-engined flying boat of at least 12,000lbs. displacement. In other words, the pioneer work up to this size was done. Porte led the way, others followed. Having the advantage of this knowledge, and the experience gained with the Porte boats, the Air Ministry in 1917 decided to build a boat of about the same displacement—leading to the P.5. The hull lines were set out by Major Linton-Hope, and a model tested at the tank, where certain modifications were suggested as the result of experience with previous models tested. The hull construction was to Linton-Hope's design and was built by Messrs. May, Harden and May. The hull was then handed over to the Phœnix Dynamo Co. to fit the superstructure, etc., which firm were then building F.3's and F.5's.

Owing to the high L.W.L. and lack of experience, the planes were set too low on the hull, and had to be considerably raised after preliminary trials had been carried out. The various hull defects have already been indicated in my paper.

In reply to Captain Hill, I regret I am unable to follow his argument on the disadvantages to be expected from a high aspect ratio, unless he is referring to manœuvrability. From the aerodynamic point of view I am inclined to think that the aspect ratio commonly used on flying boats leads to greater efficiency than that used on the corresponding aeroplanes, thus partly accounting for the better performance of the former.

The short tailplane arm makes for less damping of the longitudinal oscillations, giving a more sensitive machine fore and aft. The control appears to be quite satisfactory. I agree that allowance for sweep back is most useful, but owing to

the wing tip floats and covered-in pylons, would lead to some complication in the case of the boats under discussion.

I am not at all clear on what he means with regard to the connection between aspect ratio and factors of safety. Maybe there is an error in the reporting of his remarks.

While I have no definite data to go on, I think a vertical velocity of 15ft. per when landing is quite reasonable, and could be obtained without much sacrifice in other directions.

From the time the keel first touches the water when landing, the distance the boat sinks into the water will depend upon the displacement of the boat, the under-water lines and the vertical velocity.

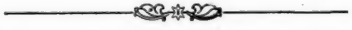
The accident to the " Fury " was certainly not the result of having insufficient or inefficient control. The control proved to be ample, and was well within the power of the pilot.

In reply to Major Buchanan, I would like it to be known that his remarks on the " F " boats carry considerable weight, because as head of the Aircraft Design Dept., Directorate of Research, Air Ministry, he is closely in touch with all the latest developments in aircraft and has an accurate knowledge of the work carried out by these boats; and also having the good fortune to be an engineer and not a naval architect, his opinions may be looked upon as unbiassed.

The weights used in compiling Table II., with the exception of those of the P.S.B. and the F.5, for which I can vouch, were taken from Air Ministry weight sheets, which I had assumed to be reasonably correct. I am still of the opinion, and feel sure future designs will show to be the case, provided tank tests are not taken too seriously, that a large flying boat can be designed to carry a useful load at least equal to, if not greater than, the corresponding aeroplane. The air performance on general considerations must be superior. With a propeller suitably designed to meet the water conditions, and light hull loading, I think it will be found unnecessary to provide greater engine weight.

While the single engine flying boat may fulfil certain Service requirements, I doubt very much if it ever will be a commercial proposition, except when operating under conditions outlined in my paper. I think Major Buchanan will agree that there is ample scope for improvement in the aerodynamic and detail design of the small boats in use at present.

In reply to Mr. Knight, the question of the cost of construction is beyond the scope of this paper. With regard to the working of the planking round the chine on the " Fury " hull, I would say this was a comparatively simple operation. The timbers were bent round the stringers in the usual way, and the inner skin, which was fore and aft, fastened on. The two diagonal skins were then laid over, steamed at the bend. The cost would not be at all prohibitive.



ON THE STABILITY OF AERO ENGINES

BY CAPT. J. MORRIS, B.A., A.F.R.AË.S.

INTRODUCTION

The question of the stability of an aero engine is one of the utmost importance. An aerodynamically unstable aeroplane does not necessarily involve failure or disaster, but a dynamically unstable engine in an aeroplane is fraught with the gravest consequences. Frequent cases of failure have undoubtedly occurred in practice through ignorance or avoidance of this important problem. At least one Zeppelin was forced to land in this country during the war through failure of a shaft by whirling or torsional resonance and considerable trouble was experienced with the early N.S. airships from this cause. According to the Report of the Advisory Committee for Aeronautics for the year 1918-1919: There have been many failures of the crankshafts in aero engines, and the question is one of some moment. . . . In certain types of engines the airscrew bosses have been found to heat and even burn.

The main problem involved in the stability of aero engines is this: The system as a whole will vibrate during rotation with certain frequencies; in addition, particular items may oscillate independently of the whole system. All possible frequencies, whether for the system or independently for members of the system, should be examined and estimated and compared with the frequency of the engine impulses. At such speeds where resonance occurs, that is, when the natural frequency of oscillation is equal to the engine impulse frequency, failure will occur if that speed is maintained. In addition, failure may occur by whirling, that is, when the speed of rotation is so great that centrifugal force overcomes the tendency of the stiffness of a member to maintain that member in vibration. Whirling is more likely to occur with long shafts supported in bearings at distances from one another; with short shafts and close bearings failure from this cause is unlikely. Failure through torsional resonance is most probably the more frequent phenomenon. The following investigation aims at envisaging the problem in a perfectly general manner. The mathematics involved will be found to be reasonably simple and use of the results only involves an elementary knowledge of algebra.

Within the author's knowledge the treatment is entirely original.

PART I.

General Principles—Resonance, Whirling—The Airscrew and the Flexibility of the Blades.

1. To illustrate the principles involved in the treatment of the stability of rotating shafts, consider a light straight shaft (not necessarily uniform in section) carrying a single concentrated load and running in any bearings.

Let the load be W_1 lbs. and let P_1 be its point of attachment.

Let y_{11} ins. be the static deflection of the shaft at P_1 due to unit load at that point, then if Y_1 is the static deflection at P_1, due to the load W_1, we have

$$Y_1 = W_1 y_{11} \quad . \quad . \quad . \quad . \quad . \quad . \quad (1)$$

Now let W_1 be pulled aside and then let go, the subsequent motion taking place in a horizontal plane.

At time t let Y_1 be the lateral displacement of the load W_1.

Its acceleration will be

d^2Y_1/dt^2 or $\ddot{Y}_1$ radially outwards.

The inertia force at P_1 is

$$-W_1\ddot{Y}_1/g \quad . \quad . \quad . \quad . \quad . \quad . \quad (2)$$

Hence the equation of motion of W_1 will be

$$Y_1 = -(W_1\ddot{Y}_1/g)\,y_{11} \quad . \quad . \quad . \quad . \quad . \quad (3)$$

or

$$(m_1y_{11}D^2 + 1)\,Y_1 = 0 \quad . \quad . \quad . \quad . \quad . \quad (4)$$

where $m_1 = W_1/g$, g being in ins./sec.^{2} and D is the operator d/dt.

The motion is *simple harmonic*.

The frequency N_v—the number of double vibrations per second—will be given by $k/2\pi$ where

$$k^2 = 1/m_1y_{11} \quad . \quad . \quad . \quad . \quad . \quad (5)$$

Hence

$$N_v = (1/2\pi)\sqrt{(g/W_1y_{11})} \quad . \quad . \quad . \quad . \quad . \quad (6)$$

Now let the shaft rotate as well as vibrate.

If Y_1 is the deflection of the shaft at P_1, the two component accelerations of W_1 will be:

(i.) $\ddot{Y}_1 - \Omega^2Y_1$ radially outwards and

(ii.) $2\Omega\dot{Y}_1$ at right angles to (i.),

where Ω is the angular velocity of rotation of the shaft.

$\ddot{Y}_1 - \Omega^2Y_1$ will in general be large compared with $2\Omega\dot{Y}_1$.

We will suppose that the shaft is bent in one plane.

The force bending the shaft is

$$-W_1(\ddot{Y}_1 - \Omega^2Y_1)/g + W_1\cos\Omega t \quad . \quad . \quad . \quad (7)$$

The first member of (7) being an inertia force and the second the component of the weight.

The equation of motion of W_1 will be

$$Y_1 = [-W_1(\ddot{Y}_1 - \Omega^2Y_1)/g + W_1\cos\Omega t]\,y_{11} \quad . \quad . \quad (8)$$

or

$$(D^2 + g/W_1y_{11} - \Omega^2)\,Y_1 = g\cos\Omega t \quad . \quad . \quad . \quad (9)$$

Thus

$$Y_1 = g\cos\Omega t/(g/W_1y_{11} - 2\Omega^2) + A\cos[\sqrt{(g/W_1y_{11} - \Omega^2)}\,t + \epsilon] \quad . \quad (10)$$

where A and ϵ are arbitrary constants.

The motion of W_1 is made up of two harmonics:

(i.) A forced vibration of frequency $\Omega/2\pi$ due to the revolution of the weight.

(ii.) A vibration of frequency $(1/2\pi)\sqrt{(g/W_1y_{11} - \Omega^2)}$ due to the centrifugal force of W_1.

Instability will arise when

$$2\Omega^2 = g/W_1y_{11} \quad . \quad . \quad . \quad . \quad . \quad (11)$$

At this speed the amplitude of the forced vibration will become infinite. It occurs when

$$\Omega/2\pi = (1/2\pi)\sqrt{(g/W_1y_{11} - \Omega^2)} \quad . \quad . \quad . \quad (12)$$

i.e., when the frequency of the forced vibration due to the revolution of the weight coincides with the natural frequency of the shaft when rotating with angular velocity Ω. Such a speed is called a *critical speed of resonance*.

Instability will also arise when

$$\Omega^2 = g/W_1 y_{11} \quad . \quad . \quad . \quad . \quad . \quad (13)$$

At this speed the centrifugal force of the load will overcome the stiffness of the shaft and vibration will cease, the displacement tending to increase indefinitely.

Such a speed is called a *critical speed of whirling*.

In the above case whirling takes place when

$$\Omega/2\pi = (1/2\pi)\sqrt{(g/W_1 y_{11})} \quad . \quad . \quad . \quad . \quad (14)$$

i.e., when the frequency of rotation of the shaft—the number of revolutions per second—coincides with the frequency of vibration of W_1, when the shaft is not rotating.

It should be observed that if the shaft is vertical there will not be a forced vibration due to the weight of W_1 and consequently there will be no critical speed of resonance, although the critical speed of whirling will be unaffected.

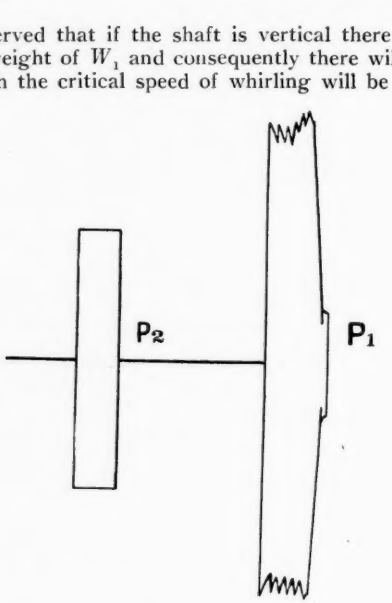


FIG. 1.

2. Consider next an airscrew P_1 driven by a gear wheel P_2 (see Fig. 1). P_2 is subjected to an applied torque T_2 and T_1 is the air torque opposing the rotation of P_1. There are supposed to be bearings in the neighbourhood of both P_1 and P_2, the frictional torques being t_1 and t_2 respectively.

At time t let θ_1 θ_2 be the angular displacements of P_1 and P_2 respectively. Then the equations of motion of P_1 and P_2 are

$$p_1\ddot{\theta}_1 = c_{12}(\theta_2 - \theta_1) - (T_1 + t_1) \quad . \quad . \quad . \quad . \quad (1)$$

$$p_2\ddot{\theta}_2 = T_2 - t_2 - c_{12}(\theta_2 - \theta_1) \quad . \quad . \quad . \quad . \quad (2)$$

where $p_1 = I_1/g$, $p_2 = I_2/g$; I_1 and I_2 being the moments of inertia of P_1 and P_2 about the axis of the shaft; c_{12} is the torsional stiffness of the shaft between P_1 and P_2 ($= CI/L$ for a uniform shaft, L being its length, I the polar moment of inertia of its cross section and C its modulus of rigidity).

From (1) and (2) we obtain

$$\ddot{\theta}_{12} = -c_{12}(1/p_1 + 1/p_2)\theta_{12} + (T_1 + t_1)/p_1 + (T_2 - t_2)/p_2 \quad . \quad (3)$$

where

$$\theta_{12} = \theta_2 - \theta_1$$

Thus

$$[D^2 + c_{12}(1/p_1 + 1/p_2)]\theta_{12} = (T_1 + t_1)/p_1 + (T_2 - t_2)/p_2 \quad . \quad (4)$$

If T_1, T_2, t_1, t_2, are constant we find from (4)

$$\theta_{12} = [(T_1 + t_1)p_1 + (T_2 - t_2)p_2]/c_{12}(1/p_1 + 1/p_2) + A\cos(kt + \epsilon) \quad . \quad (5)$$

where

$$k^2 = c_{12}(1/p_1 + 1/p_2)$$

and A and ϵ are arbitrary constants. In this case T_1, T_2, t_1, t_2, only affect the mean angle between P_1 and P_2 which remains constant. They do not affect the period of vibration which is the same as the free period when the system is not power driven.

If the mean motion is uniform

$$T_2 - t_2 - T_1 - t_1 = 0$$

so that

$$\theta_{12} = (T_2 - t_2)/c_{12} + A\cos(kt + \epsilon) \quad . \quad . \quad . \quad (6)$$

In an internal combustion engine, however, T_2 (and consequently T_1) is not constant. T_2 and T_1 in general are periodic functions involving a series of harmonics, the fundamental frequency being the number of firing impulses per second.

If N is the number of revolutions per minute of the crankshaft and n the number of cylinders, the frequency of the firing impulses is

$$nN/120 = f/2\pi \text{ (say)} \quad . \quad . \quad . \quad . \quad . \quad (7)$$

$(T_1 + t_1)/p_1 + (T_2 - t_2)/p_2$ will in general consist of a Fourier series

$$T + \sum_{q=1}^{q=\infty} (a_q \cos qft + b_q \sin qft) \quad . \quad . \quad . \quad . \quad (8)$$

where T, a_q, b_q, are constant at any particular speed.

Equation (4) under these conditions becomes

$$(D^2 + k^2)\theta_{12} = T + \sum_{q=1}^{q=\infty} (a_q \cos qft + b_q \sin qft) \quad . \quad . \quad (9)$$

The solution of which is

$$\theta_{12} = T/k^2 + \sum_{q=1}^{q=\infty} (a_q \cos qft + b_q \sin qft)/(k^2 - q^2f^2) + A\cos(kt + \epsilon) \quad (10)$$

From (10) we observe that

(i.) There will be imposed upon the system a series of forced vibrations, viz.

$$\sum_{q=1}^{q=\infty} (a_q \cos qft + b_q \sin qft)/(k^2 - q^2f^2) \quad . \quad . \quad . \quad (11)$$

(ii.) The system will have a natural frequency

$$k/2\pi = (1/2\pi)\sqrt{[c_{12}(1/p_1 + 1/p_2)]} \quad . \quad . \quad . \quad (12)$$

From (II.) we note that the angular difference between P_2 and P_1 will involve an harmonic of infinite amplitude when

$$k^2 = q^2 f^2 \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad (13)$$

or

$$k/2\pi = qf/2\pi \quad . \quad . \quad . \quad . \quad . \quad . \quad (14)$$

i.e., when the natural frequency of vibration of the system is an exact multiple of the frequency of the firing impulses.

At such an engine speed we have instability due to *torsional resonance* and the speed at which it occurs is called the critical speed of torsional resonance.

In the case considered (14) can be written

$$(1/2\pi)\sqrt{[c_{12}(1/p_1 + 1/p_2)]} = qnN/120 \quad . \quad . \quad . \quad (15)$$

N being the r.p.m. of the crankshaft, n the number of cylinders and q a whole number.

If p_1 is large and p_2 is relatively small a first approximation to (15) is

$$(1/2\pi)\sqrt{(c_{12}/p_2)} = qnN/120 \quad . \quad . \quad . \quad . \quad (16)$$

In addition the forces due to reciprocating parts and gravity will be periodic, the frequency being

$$\Omega/2\pi \text{ or } N/60 \quad . \quad . \quad . \quad . \quad . \quad (17)$$

where Ω is the angular velocity of the system and N the corresponding r.p.m.

Thus resonance will occur when

$$k/2\pi = qN/60 \quad . \quad . \quad . \quad . \quad . \quad (18)$$

where $k/2\pi$ is the frequency of the system or an individual member of the system.

The importance of values of q above unity in the case of (15) can only be assessed by experiment. Failure will certainly occur when $q = 1$ and vibration may be violent when $q = 2$, becoming less marked as q increases.

It remains to try out by experiment the resonance as indicated by (18).

3. In the preceding problem the airscrew was regarded as a rigid body. This, of course, is not the case in practice. In general the blades of an airscrew are flexible and this flexibility may have an appreciable influence on the periods of vibration of a system of which an airscrew is a part. Owing to the complexity of the section of an airscrew it would prove extremely difficult to estimate this effect by analysis.

If the blades could be regarded as light flexible arms carrying loads at the tips the problem would be greatly simplified. In order to investigate the possibility of an actual airscrew being replaced by such a system consider first the following problem (see Fig. 2):

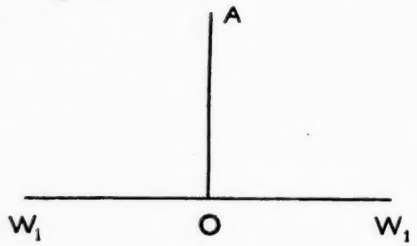


FIG. 2.

AO is a vertical bar fixed at A. W_1 O W_1 is a pair of equal light flexible arms carrying tip loads W_1. The arms are displaced in a horizontal plane and then released so that the system on the whole executes pure torsional oscillations about AO.

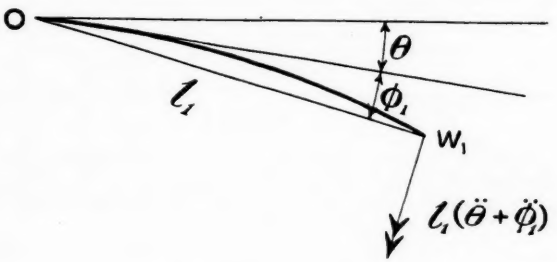


FIG. 3.

Fig. 3 is a plan view of one of the displaced arms. θ is the angular displacement of the normal line of the arms and ϕ_1 the angular displacement of W_1 due to the flexibility of the arm.

The acceleration of W_1 as shown is

$$l_1(\ddot{\theta}+\ddot{\phi}_1) \quad . \quad . \quad . \quad . \quad . \quad . \quad (1)$$

Thus the arm regarded as a cantilever has a deflection $l_1\phi_1$ due to an inertia force

$$-W_1 l_1(\ddot{\theta}+\ddot{\phi}_1)/g \quad . \quad . \quad . \quad . \quad . \quad (2)$$

Hence the equation of motion of W_1 is

$$l_1\phi_1 = -W_1 l_1(\ddot{\theta}+\ddot{\phi}_1)\, y_{11}/g \quad . \quad . \quad . \quad . \quad (3)$$

where y_{11} is the deflection of the arm, regarded as cantilever, due to unit load at rest.

Similarly for the other arm

$$l_1\phi_2 = -W_1 l_1(\ddot{\theta}+\ddot{\phi}_2)\, y_{11}/g \quad . \quad . \quad . \quad . \quad (4)$$

Also by moments about the axis of OA

$$-W_1 l_1^2(\ddot{\theta}+\ddot{\phi}_1)/g - W_1 l_1^2(\ddot{\theta}+\ddot{\phi}_2)/g = c_{12}\theta \quad . \quad . \quad (5)$$

where $c_{12} = CI/L$ for the bar.

Writing

$$c_1 = l_1^2/y_{11} \quad . \quad . \quad . \quad . \quad . \quad . \quad (6)$$

and

$$p_1 = W_1 l_1^2/g \quad . \quad . \quad . \quad . \quad . \quad (7)$$

(3), (4) and (5) can be written

$$c_1\phi_1 = -p_1(\ddot{\theta}+\ddot{\phi}_1) \quad . \quad . \quad . \quad . \quad . \quad (8)$$

$$c_1\phi_2 = -p_1(\ddot{\theta}+\ddot{\phi}_2) \quad . \quad . \quad . \quad . \quad . \quad (9)$$

or

$$(D^2/c_1 + 1/p_1)\,[c_1\phi_1 \text{ or } c_1\phi_2] = -D^2\theta \quad . \quad . \quad . \quad (10)$$

and

$$c_1(\phi_1+\phi_2) = c_{12}\theta \quad . \quad . \quad . \quad . \quad . \quad (11)$$

will be the bending moment at O.

From (10) we find

$$(D^2/c_1 + 1/p_1)\, c_1(\phi_1-\phi_2) = 0 \quad . \quad . \quad . \quad (12)$$

Thus one of the frequencies of the arms will be

$$(1/2\pi)\sqrt{(c_1/p_1)} \quad . \quad . \quad . \quad . \quad . \quad (13)$$

which is the same as that for the free vibration of one of these arms.

From (10) and (11) we find

$$2D^2\theta + (D^2/c_1 + 1/p_1)\, c_{12}\theta = 0 \quad . \quad . \quad . \quad (14)$$

or

$$[(1/c_{12} + 1/2c_1)\, D^2 + 1/2p_1]\, \theta = 0 \quad . \quad . \quad . \quad (15)$$

From which we obtain the frequency

$$1/2\pi\sqrt{[2p_1\,(1/c_{12} + 1/2c_1)]} \quad . \quad . \quad . \quad . \quad (16)$$

4. Next let the arms be replaced by an actual airscrew. Let θ be the displacement of the normal line of the airscrew blades and let y_1 be the displacement from the normal line of the airscrew blades of an element dx on one side, at a distance x from the centre line of the vertical bar.

The acceleration of this element will be

$$d^2y_1/dt^2 + x\,(d^2\theta/dt^2) \quad . \quad . \quad . \quad . \quad . \quad (1)$$

so that the equation of motion will be

$$(d^2/dx^2)\,(EI_x\, d^2y_1/dx^2) = -(\rho A_x/g)\,(d^2y_1/dt^2 + x\, d^2\theta/dt^2) \quad . \quad (2)$$

where A_x is the area of cross section at the element, I_x the appropriate moment of inertia of this section, ρ the density of the material and E its Young's modulus. Similarly for the complementary element on the other blade

$$(d^2/dx^2)\,(EI_x\, d^2y_2/dt^2) = -(\rho A_x/g)\,(d^2y_2/dt^2 + x\, d^2\theta/dt^2) \quad . \quad (3)$$

From (2) and (3) we have

$$(d^2/dx^2)\,(EI_x\, d^2y/dx^2) = -(\rho A_x/g)\, d^2y/dt^2 \quad . \quad . \quad . \quad (4)$$

where

$$y = y_1 - y_2 \quad . \quad . \quad . \quad . \quad . \quad (5)$$

The solution of (4) will be the same as that for the free vibration of one of the blades. Hence the blades will have in addition to any others the frequencies which each would have in free vibration.

If the blades were of uniform section (4) becomes

$$EI\,(d^4y/dx^4) = -(w/g)\, d^2y/dt^2 \quad . \quad . \quad . \quad . \quad (6)$$

where w is the weight per unit length.

Let

$$y = XT$$

where X is a function of x only and T a function of t only. Then (6) becomes

$$EI\,(d^4X/dx^4)/X = -(w/g)\,(d^2T/dt^2)/T \quad . \quad . \quad . \quad (7)$$

Let now

$$d^4X/dx^4 = \lambda^4 X \quad . \quad . \quad . \quad . \quad . \quad (8)$$

and

$$d^2T/dt^2 = -k^2T \quad . \quad . \quad . \quad . \quad . \quad (9)$$

so that

$$EI\lambda^4 = wk^2/g \quad . \quad . \quad . \quad . \quad . \quad (10)$$

and

$$y = \cos(kt + \epsilon)\,[\alpha \cosh \lambda x + \beta \sinh \lambda x + \gamma \cos \lambda x + \delta \sin \lambda x] \quad . \quad (11)$$

where α, β, γ, δ and ϵ are arbitrary constants.

When $x = 0$, $y_1 = 0$, $y_2 = 0$; $dy_1/dx = 0$, $dy_2/dx = 0$ for all t so that

$$\alpha + \gamma = 0 \quad . \quad . \quad . \quad . \quad . \quad (12)$$

and

$$\beta + \delta = 0 \quad . \quad . \quad . \quad . \quad . \quad (13)$$

When $x = l_1$, $d^2y_1/dx^2 = 0$, $d^2y_2/dx^2 = 0$; $d^3y_1/dx^3 = 0$, $d^3y_2/dx^3 = 0$ for all t so that

$$\alpha(\cosh \lambda l_1 + \cos \lambda l_1) + \beta(\sinh \lambda l_1 + \sin \lambda l_1) = 0 \quad . \quad . \quad (14)$$

and

$$\alpha(\sinh \lambda l_1 - \sin \lambda l_1) + \beta(\cosh \lambda l_1 + \cos \lambda l_1) = 0 \quad . \quad . \quad (15)$$

From (14) and (15) we find

$$\cosh \lambda l_1 \cos \lambda l_1 + 1 = 0 \quad . \quad . \quad . \quad . \quad . \quad (16)$$

The lowest value of λl_1 which satisfies (16) is

$$\lambda l_1 = 1.8731 \quad . \quad . \quad . \quad . \quad . \quad (17)$$

or

$$\lambda^4 l_1^4 = 12.46 \quad . \quad . \quad . \quad . \quad . \quad (18)$$

Now

$$EI\lambda^4 = wk^2/g \quad . \quad . \quad . \quad . \quad . \quad (19)$$

$$\therefore\ k^2 = 12.46\, EIg/wl_1^4 \quad . \quad . \quad . \quad . \quad (20)$$

or

$$k^2 = (3EI/l_1)/(Wl_1^2/4.11\, g) \quad . \quad . \quad . \quad . \quad (21)$$

where $W = wl_1$, the weight of one blade.

Thus

$$k^2 = c_1/p_1 \quad . \quad . \quad . \quad . \quad . \quad (22)$$

where

$$c_1 = 3EI/l_1 \text{ and } p_1 = Wl_1^2/4.11\, g$$

A sufficiently close approximation is

$$k^2 = 4gc_1/Wl_1^2 \quad . \quad . \quad . \quad . \quad . \quad (23)$$

Hence the equivalent tip load for free vibration of the blades when of uniform section is $W/4$.

Unfortunately the tip load for free vibration does not necessarily give the correct bending moment at the root and in an endeavour to overcome this difficulty we will investigate this bending moment.

The equations of motion of the blades are

$$EId^4(y_1 \text{ or } y_2)/dx^4 = -(w/g)[d^2(y_1 \text{ or } y_2)/dt^2 + xd^2\theta/dt^2] \quad . \quad (24)$$

or

$$EI(d^4y/dx^4) = -(w/g)[d^2y/dt^2 + 2xd^2\theta/dt^2] \quad . \quad . \quad (25)$$

where

$$y = y_1 + y_2$$

A solution of (25) is

$$y + 2x\theta = \cos(kt + \epsilon)[\alpha \cosh \lambda x + \beta \sinh \lambda x + \gamma \cos \lambda x + \delta \sin \lambda x] \quad . \quad (26)$$

where

$$EI\lambda^4 = wk^2/g$$

When $x = 0$, $y_1 = 0$, $y_2 = 0$; $dy_1/dx = 0$, $dy_2/dx = 0$ for all t so that

$$\alpha + \gamma = 0 \quad . \quad . \quad . \quad . \quad . \quad (27)$$

and

$$2\theta = (\beta + \delta)\lambda \cos(kt + \epsilon) \quad . \quad . \quad . \quad . \quad (28)$$

When $x = l_1$, $d^2y_1/dx^2 = 0$, $d^2y_2/dx^2 = 0$; $d^3y_1/dx^3 = 0$, $d^3y_2/dx^3 = 0$ for all t so that

$$\alpha(\cosh \lambda l_1 + \cos \lambda l_1) + \beta \sinh \lambda l_1 - \delta \sin \lambda l_1 = 0 \quad . \quad . \quad (29)$$

and

$$\alpha(\sinh \lambda l_1 - \sin \lambda l_1) + \beta \cosh \lambda l_1 - \delta \cos \lambda l_1 = 0 \quad . \quad . \quad (30)$$

Thus

$$2\theta = 2\alpha \cos(kt + \epsilon)[\lambda \cosh \lambda l_1 \cos \lambda l_1 + 1]/(\cosh \lambda l_1 \sin \lambda l_1 - \cos \lambda l_1 \sinh \lambda l_1) \quad (31)$$

Also by moments

$$EI\,(d^2y_1/dx^2)_{x=0} + EI\,(d^2y_2/dx^2)_{x=0} = c_{12}\theta \quad . \quad . \quad (32)$$

or

$$EI\,(d^2y/dx^2)_{x=0} = c_{12}\theta \quad . \quad . \quad . \quad . \quad (33)$$

the c_{12} referring to the bar, or

$$2EI\alpha\lambda^2 \cos(kt + \epsilon) = c_{12}\theta \quad . \quad . \quad . \quad . \quad (34)$$

Hence from (31) and (34) we find

$$2c_0/3c_{12} = (\cosh \lambda l_1 \cos \lambda l_1 + 1)/\lambda l_1\,(\cosh \lambda l_1 \sin \lambda l_1 - \cos \lambda l_1 \sinh \lambda l_1) \quad (35)$$

where

$$c_0 = 3EI/l_1 \text{ and } c_{12} = CI/L$$

the former referring to the blades and the latter to the bar.

As a first approximation we find

$$1/c_0\lambda^4 l_1^4 = (2/9)\,[(1/c_{12}) + (99/140)\,(1/2c_0)] \quad . \quad . \quad (36)$$

and since

$$EI\lambda^4 = wk^2/g$$

we have approximately

$$k^2 = 3g/8W_0l_1^2\,(1/c_{12} + 3/8c_0) \quad . \quad . \quad . \quad (37)$$

where W_0 is the equivalent tip load of one blade for free vibration (*i.e.*, $W/4$, where W is the weight of the blade).

If the blades were light, of flexural coefficients c_1 and had tip loads W_1.

$$k^2 = g/2W_1l_1^2\,(1/c_{12} + 1/2c_1) \quad . \quad . \quad . \quad . \quad (38)$$

Thus, comparing (37) with (38), the frequencies with the uniform loading will be approximately the same as if the blades were light, the tip being loaded with $1/3$ the weight of each blade, or $4/3$ the equivalent tip load in free vibration and c_0 increased in the same proportion $4/3$ times. It should be observed that the tip loads $W/3$ will give the same moment of inertia of the airscrew about its axis of revolution as its own weight.

5. From §3 and §4 the treatment suggested in the case of an actual airscrew is as follows:—

Let W_0 be the equivalent tip load for one blade in free vibration and let c_0 be its flexural coefficient. Next let W_0 and c_0 be increased in the same ratio to W_1 and c_1 so that $W_1/W_0 = c_1/c_0$ is the necessary factor in order that the bending moment at the root of the blade should be correct. This increase will not affect the period of free vibration. The blade is thus reduced to a light arm of known stiffness loaded at the tip.

Similarly, quantities are found for the case when the blades vibrate in a fore and aft direction.

These quantities can be found by experiment, but before indicating an experimental method for finding them we will investigate the bending of a light cantilever of uniform section under two end loads, one vertical and the other inclined at a small angle to the horizon (see Fig. 4). The object of this investigation is to examine how the centrifugal force of the equivalent tip loads influences the main problem under consideration.

RP is the cantilever bent under the vertical load W and the load F at an angle ψ to RQ. The line of action of F meets the vertical at R at the point S, so that $RS = \xi$ and $RQS = \theta$, PQ being vertical and $= \eta$.

N.B.—P is the point where the lines of action of W and F meet.

If l is the length of the cantilever

$$l\psi = \xi + \eta \quad . \quad . \quad . \quad . \quad . \quad (1)$$

The bending moment at a distance x from R the point of fixture is

$$EI\,(d^2y/dx^2) = (W + F\psi)\,(l - x) - F\,(\eta - y) \quad . \quad . \quad (2)$$

or

$$EI\,(d^2y/dx^2) - Fy = (W + F\psi)\,(l - x) - F\eta \quad . \quad . \quad (3)$$

where EI is the flexural rigidity of the cantilever.

$$\therefore\ y = A \sinh \sqrt{(F/EI)}\, x + B \cosh \sqrt{(F/EI)}\, x - (W/F + \psi)\,(l - x) + \eta \quad (4)$$

When

$$x = 0,\ y = 0\ \therefore\ B - (Wl/F + \xi) = 0 \quad . \quad . \quad . \quad (5)$$

when

$$x = 0,\ dy/dx = 0\ \therefore\ A\sqrt{(F/EI)} + [W/F + (\xi + \eta)/l] = 0 \quad . \quad (6)$$

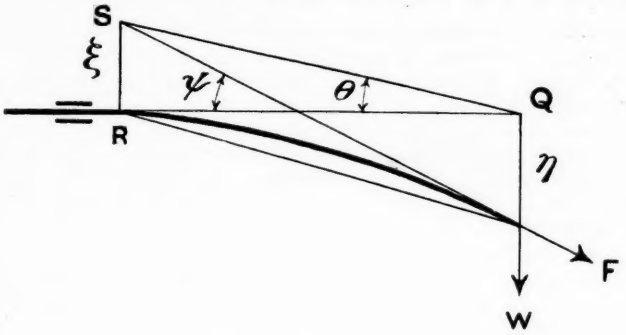


FIG. 4.

Also when $x = l,\ y = \eta$

$$\therefore\ \eta = -[W/F + (\xi + \eta)/l]\sqrt{(EI/F)} \sinh \sqrt{(F/EI)}\, l + (Wl/F + \xi) \cosh \sqrt{(F/EI)}\, l + \eta \quad . \quad . \quad . \quad (7)$$

leading to

$$\eta = (W + F\theta)\,\Phi\,(\alpha)\, l^3/3EI \quad . \quad . \quad . \quad . \quad (8)$$

where $\Phi\,(\alpha)$ is the Berry function,

$$3\,(\alpha \coth \alpha - 1)/\alpha^2 \quad . \quad . \quad . \quad . \quad . \quad (9)$$

and

$$\alpha = \sqrt{(F/EI)}\, l \quad . \quad . \quad . \quad . \quad . \quad (10)$$

When F is upwardly inclined θ will be negative.

It is to be observed that when α is small, *i.e.*, when Fl^2/EI is small, $\Phi\,(\alpha)$ will be practically unity.

In cases which we shall deal with F will usually be centrifugal force and having regard to the practical speeds and the stiffness of the members considered α will in general be small, so that approximately

$$\eta = (W + F\theta)\, l^3/3EI \quad . \quad . \quad . \quad . \quad (11)$$

The fixing couple will be

$$(W + F\theta)\, l \quad . \quad . \quad . \quad . \quad . \quad (12)$$

We also find that the slope at P is

$$(W + F\theta)\,\Psi\,(\alpha)\,(l^2/2EI) \quad . \quad . \quad . \quad . \quad (13)$$

where

$$\Psi\,(\alpha) = (\tanh \tfrac{1}{2}\alpha)/\tfrac{1}{2}\alpha \quad . \quad . \quad . \quad . \quad . \quad (14)$$

and which is practically unity when α is small.

If F was absent the deflection and slope at P, due to the load W, would be $Wl^3/3EI$, $Wl^2/2EI$, respectively.

Thus the effect of the lateral load F on the deflection and slope of the shaft at P is the same as if W were increased to $W + F\theta$ and there was no lateral load.

When $\theta = 0$, *i.e.*, when the line of action of F passes through the point of fixture of the cantilever the deflection is

$$Wl^3/3EI \quad . \quad . \quad . \quad . \quad . \quad . \quad (15)$$

approximately, the slope is

$$Wl^2/2EI \quad . \quad . \quad . \quad . \quad . \quad . \quad (16)$$

approximately, and the fixing couple is

$$Wl \quad . \quad . \quad . \quad . \quad . \quad . \quad (17)$$

Under these circumstances the lateral load has practically no influence on the deflection of the shaft due to the vertical load.

6. We will now consider an experimental determination of the equivalent tip loads and flexural coefficients of the blades of an airscrew.

Let the airscrew be attached to one end of a straight uniform bar of circular section, the other end of the bar being fixed. In the normal position the bar is vertical and the plane of rotation of the airscrew is horizontal.

The system is given a torsional displacement about the axis of the vertical bar and then let go so as to execute, as far as can be observed, pure torsional oscillations. The bar is of such length that the periods of these oscillations can be readily measured.

Suppose in the first case the airscrew is two-bladed.

Let W_1 be the equivalent tip load and c_1 the equivalent flexural coefficient for each blade. We have already proved in §3 that such a system will have frequencies as follows:

(i.) For the system

$$1/2\pi\sqrt{[2p_1(1/c_{12} + 1/2c_1)]} \quad . \quad . \quad . \quad . \quad (1)$$

and (ii.) for the blades in addition

$$(1/2\pi)\sqrt{(c_1/p_1)} \quad . \quad . \quad . \quad . \quad . \quad (2)$$

(i.) will be the fundamental frequency for the system with the prearranged conditions.

If T be the period of oscillation of the fundamental

$$T^2/4\pi^2 = 2p_1(1/c_{12} + 1/2c_1) \quad . \quad . \quad . \quad . \quad (3)$$

If $T^2/4\pi^2$ be plotted against $1/c_{12}$ we obtain a straight line. The slope of this line will be

$$2p_1 \text{ or } 2W_1l_1^2/g \quad . \quad . \quad . \quad . \quad . \quad (4)$$

and the intercept in the $1/c_{12}$ axis will be

$$1/2c_1 \quad . \quad . \quad . \quad . \quad . \quad . \quad (5)$$

Thus we find both p_1 and c_1 and consequently we know the other frequency of the blades which is $k/2\pi$ where

$$k^2 = c_1/p_1 \quad . \quad . \quad . \quad . \quad . \quad . \quad (6)$$

Now c_0, the actual flexural coefficient of a blade, can be found by direct measurement. Suppose a load W, attached at the tip of one blade bent in the plane of rotation of the airscrew, produces a deflection d_1 then

$$c_0 = Wl_1^2/d_1 \quad . \quad . \quad . \quad . \quad . \quad (7)$$

Since $W_0/W_1 = c_0/c_1$ we also have W_0 the actual equivalent tip load in free vibration.

In the case of a four-bladed airscrew we find for the tip loads

$$(D^2/c_1 + 1/p_1)\,c_1\phi_r\ (r = 1, 2, 3 \text{ or } 4) = -D^2\theta \quad . \quad . \quad (8)$$

and by moments

$$c_1(\phi_1 + \phi_2 + \phi_3 + \phi_4) = c_{12}\theta \quad . \quad . \quad . \quad . \quad (9)$$

From which we obtain

(i.) A frequency of the system

$$1/2\pi\sqrt{[4p_1(1/c_{12} + 1/4c_1)]} \quad . \quad . \quad . \quad . \quad (10)$$

(ii.) A blade frequency in addition

$$(1/2\pi)\sqrt{(c_1/p_1)} \quad . \quad . \quad . \quad . \quad . \quad (11)$$

Thus

$$T^2/4\pi^2 = 4p_1\,(1/c_{12} + 1/4c_1) \quad . \quad . \quad . \quad . \quad (12)$$

From which we can obtain p_1 and c_1 as before.

7. Next, to ascertain W'_1 and c'_1 the equivalent tip load and flexural coefficient of each blade in planes perpendicular to that of rotation of the airscrew.

A two-bladed airscrew could be turned through 90° about the original axis of the blades and the same procedure as in §6 will give the quantities required.

This, however, may not be convenient and is certainly impracticable in the case of a four-bladed airscrew.

The following method is suggested:

Let the system be so displaced that the bar and the blades vibrate in the vertical plane, the oscillation on the whole being lateral. At time t let Y_1 be the displacement of the mass centre of the airscrew from the vertical and let Φ_1 be the slope of the bar at that point (see Fig. 5).

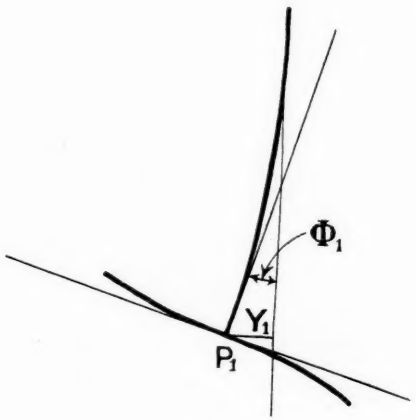


FIG. 5.

Let ϕ'_1, ϕ'_2, be the angular displacements of the tip loads due to the bending of the blades.

For the equations of motion of the tip loads we have

$$(D^2/c'_1 + 1/p'_1)[c'_1\phi'_1 \text{ or } c'_1\phi'_2] = -D^2\Phi_1 \quad . \quad . \quad . \quad (1)$$

where

$$p'_1 = W'_1 l_1^2/g$$

There will be acting on the bar at P_1

(i.) An inertia force

$$-2W\ddot{Y}_1/g \text{ or } -2m\ddot{Y}_1 \quad . \quad . \quad . \quad . \quad . \quad (2)$$

where $m = W/g$, W being the weight of each blade.

(ii.) An inertia couple

$$c'_1\,(\phi'_1 + \phi'_2) \quad . \quad . \quad . \quad . \quad . \quad (3)$$

Hence for the bar

$$Y_1 = -2m\ddot{Y}_1 y_{11} + c'_1(\phi'_1 + \phi'_2)z_{11} \quad . \quad . \quad . \quad (4)$$

$$\Phi_1 = -2m\ddot{Y}_1 \psi_{11} + c'_1(\phi'_1 + \phi'_2)\phi_{11} \quad . \quad . \quad . \quad (5)$$

where y_{11} = displacement of P_1 due to unit load at rest.

z_{11} = displacement of P_1 due to unit couple at rest.

ψ_{11} = slope at P_1 due to unit load at rest.

and ϕ_{11} = slope at P_1 due to unit couple at rest.

From (4) and (5) we have

$$Y_1\phi_{11} - \Phi_1 z_{11} = -2m\Delta_1\ddot{Y}_1 \quad . \quad . \quad . \quad . \quad (6)$$

or

$$(2m\Delta_1 D^2 + \phi_{11})Y_1 = \Phi_1 z_{11} \quad . \quad . \quad . \quad . \quad (7)$$

where

$$\Delta_1 = y_{11}\phi_{11} - z_{11}\psi_{11}$$

Also from (1) and (4) we obtain

$$(D^2/c'_1 + 1/p'_1)(2my_{11}D^2 + 1)Y_1 = -2z_{11}D^2\Phi_1 \quad . \quad . \quad (8)$$

Thus from (7) and (8) we find

$$[2D^2(2m\Delta_1 D^2 + \phi_{11}) + (D^2/c'_1 + 1/p'_1)(2my_{11}D^2 + 1)][Y_1 \text{ or } \Phi_1] = 0 \quad (9)$$

From which we obtain two frequencies $k_1/2\pi$, $k_2/2\pi$, where k_1^2, k_2^2 are roots of the equation in k^2.

$$2m(\Delta_1 + y_{11}/2c'_1)k^4 - (\phi_{11} + 1/2c'_1 + my_{11}/p'_1)k^2 + 1/2p'_1 = 0 \quad . \quad . \quad (10)$$

or

$$(1/2p'_1)(T^2/4\pi^2)^2 - (\phi_{11} + 1/2c'_1 + my_{11}/p'_1)(T^2/4\pi^2) + 2m(\Delta_1 + y_{11}/2c'_1) = 0 \quad . \quad . \quad . \quad . \quad (11)$$

where T (the period) $= 2\pi/k$.

Now y_{11}, ϕ_{11} and Δ_1 depend on the bar and are known. Also m is known. By varying the length of the bar we can obtain mean values for T and from two appropriate relations (11) we can find p'_1 and c'_1.

In the case of a four-bladed airscrew we find

$$(1/2p'_1)(T^2/4\pi^2)^2 - (\phi_{11} + 1/2c'_1 + 2my_{11}/p'_1)(T^2/4\pi^2) + 4m(\Delta_1 + y_{11}/2c'_1) = 0 \quad . \quad . \quad . \quad . \quad (12)$$

In addition there will be the frequency

$$(1/2\pi)\sqrt{(c'_1/p'_1)}$$

c'_0 can be found by direct measurement and

$$W'_0/W'_1 = c'_0/c'_1$$

where W'_0 is the actual equivalent tip load for free vibration in the direction considered.

For a uniform bar used as a cantilever as in the above experiments

$$y_{11} = L^3/3EI$$

$$z_{11} = \psi_{11} = L^2/2EI$$

$$\phi_{11} = L/EI$$

and

$$\Delta_1 = y_{11}\phi_{11} - z_{11}\psi_{11} = L^4/12(EI)^2$$

where L is the length of the bar and EI its flexural rigidity for bending.

8. To return to the problem of §2 in which an airscrew and gear wheel are both fixed to the same shaft, the airscrew being driven by the gear wheel.

The blades of the airscrew are regarded as light flexible arms of equivalent flexural coefficients c_1 and carrying equivalent tip loads W_1. The air loading of

the blades will be of a periodic nature of fundamental period, that of the engine period.

Fig. 6 is an end view. θ_2 is the displacement of the pulley P_2. The normal line of the blades is supposed displaced through an angle θ_1 and ϕ_1, ϕ_2 (supposing there are two blades) are the angular displacements of the equivalent tip loads from the normal lines of the blades. The air loading is taken as a series of air forces f_1, f_2, f_3, f_4, . . . at points 1, 2, 3, 4, . . . along the blade.

We have for the displacement at W_1

$$l_1\phi_1 = -W_1 l_1 (\ddot{\phi}_1 - \ddot{\theta}_1)\, y_{11}/g + f_1 y'_{11} + f_2 y'_{21} + \ldots f_n y'_{n1} + \ldots \quad . \quad . \quad (1)$$

Where y'_{11} is the displacement at point 1, due to unit static load, and y'_{n1} is the displacement at the point 1, due to unit static load, at the point n, the blade

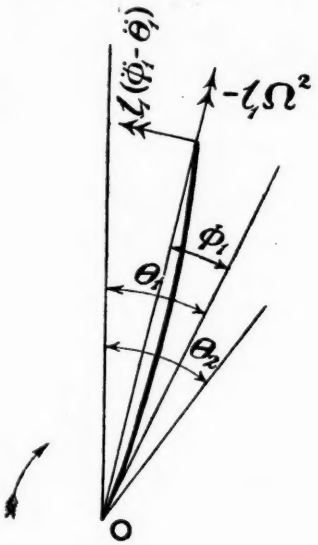


FIG. 6.

being regarded as a cantilever; and y_{11} is the displacement at point 1, due to unit static load, of the equivalent blade.

Thus (1) can be written

$$c_1\phi_1 = -p_1 (\ddot{\phi}_1 - \ddot{\theta}_1) + (l_1/y_{11})\, \Sigma f_n y'_{n1} \quad . \quad . \quad . \quad . \quad . \quad (2)$$

Now the y''s on the right hand side of (2) are elastic constants depending on the stiffness of the blade and the f's are periodic functions of fundamental frequency, that of the engine impulses.

Hence

$$c_1\phi_1 = -p_1 (\ddot{\phi}_1 - \ddot{\theta}_1) + T'_1 \quad . \quad . \quad . \quad . \quad (3)$$

or

$$(D^2/c_1 + 1/p_1)\, c_1\phi_1 = D^2\theta_1 + T'_1/p_1 \quad . \quad . \quad . \quad (4)$$

where T'_1 is a Fourier's series.

Similarly

$$(D^2/c_1 + 1/p_1)\, c_1\phi_2 = D^2\theta_1 + T''_1/p_1 \quad . \quad . \quad . \quad (5)$$

We have taken T''_1 different from T'_1 to allow for any inequality of the blades in motion.

Also by moments

$$p_1(\ddot{\phi}_1 - \ddot{\theta}_1) + p_1(\ddot{\phi}_2 - \ddot{\theta}_1) = T_1 - c_{12}(\theta_2 - \theta_1) \quad . \quad . \quad (6)$$

where T_1 is the nett air torque and c_{12} is the coefficient of torsional flexure of the shaft between the airscrew and P_2.

Using (4) and (5), (6) can be written

$$-c_1(\phi_1 + \phi_2) + T'_1 + T''_1 = T_1 - c_{12}\theta_{12} \quad . \quad . \quad . \quad (7)$$

or

$$-c_1(\phi_1 + \phi_2) + c_{12}\theta_{12} = T_1 - (T'_1 + T''_1) = T'''_1 \text{ (say)} \quad . \quad (8)$$

where

$$\theta_{12} = \theta_2 - \theta_1$$

Hence

$$-(D^2/c_1 + 1/p_1)\,c_1(\phi_1 + \phi_2) + c_{12}(D^2/c_1 + 1/p_1)\,\theta_{12} = (D^2/c_1 + 1/p_1)\,T'''_1 \quad (9)$$

and by the use of (4) and (5)

$$-2D^2\theta_1 - (T'_1 + T''_1)/p_1 + c_{12}(D^2/c_1 + 1/p_1)\,\theta_{12} = (D^2/c_1 + 1/p_1)\,T'''_1 \quad (10)$$

or

$$-2D^2\theta_1 + c_{12}(D^2/c_1 + 1/p_1)\,\theta_{12} = T \text{ (say)} \quad . \quad . \quad . \quad (11)$$

We have also

$$p_2\ddot{\theta}_2 = T_2 - c_{12}\theta_{12} \quad . \quad . \quad . \quad . \quad . \quad (12)$$

Hence from (11) and (12)

$$2D^2\theta_{12} + (D^2/c_1 + 1/p_1 + 2/p_2)\,c_{12}\theta_{12} = 2T_2/p_2 + T \quad . \quad (13)$$

or

$$[(1/2c_1 + 1/c_{12})\,D^2 + 1/2p_1 + 1/p_2]\,\theta_{12} = (2T_2/p_2 + T)/2c_{12}. \quad (14)$$

Hence the ordinary frequency of the system will be:

$$(1/2\pi)\sqrt{[(1/2p_1 + 1/p_2)/(1/2c_1 + 1/c_{12})]} \quad . \quad . \quad . \quad (15)$$

and torsional resonance will occur when

$$(1/2\pi)\sqrt{[(1/2p_1 + 1/p_2)/(1/2c_1 + 1/c_{12})]} = qnN/120 \quad . \quad (16)$$

From (4) and (5) we find

$$(D^2/c_1 + 1/p_1)\,c_1(\phi_1 - \phi_2) = (T'_1 - T''_1)/p_1 \quad . \quad . \quad (17)$$

From which we find that the blades will have in addition to the frequency (15) another of frequency

$$(1/2\pi)\sqrt{(c_1/p_1)} \quad . \quad . \quad . \quad . \quad . \quad (18)$$

that is, the ordinary frequency of a separate blade.

Where T'_1 is not $= T''_1$ there may be resonance when

$$(1/2\pi)\sqrt{(c_1/p_1)} = qnN/120 \quad . \quad . \quad . \quad . \quad (19)$$

The above investigation leads to the following conclusions:

Any part of the system may fail by resonance as given by (16). The airscrew, or the shaft by the resulting increasing oscillation of the airscrew blades, may fail by resonance as given by (19).

9. Assuming for the present that the airscrew shaft at the point of attachment of the airscrew maintains its original direction during rotation, each blade can vibrate in a plane perpendicular to that of rotation as shown in Fig. 7.

If ϕ'_1 is the angular displacement of one particular tip load W'_1 in the direction considered, the accelerations will be as shown in the figure.

We have for this particular blade

$$l_1\phi'_1 = [-W'_1 l_1\ddot{\phi}'_1/g - W'_1 l_1\Omega^2\phi'_1/g]\,y'_{11} + \Sigma f'_n y''_{n1} \quad . \quad . \quad (1)$$

or

$$c'_1\phi'_1 = -p'_1(D^2 + \Omega^2)\,\phi'_1 + (l_1/y'_{11})\,\Sigma f'_n y''_{n1} \quad . \quad . \quad (2)$$

where

$$c'_1 = l_1^2/y'_{11} \text{ and } p'_1 = W'_1 l_1^2/g$$

and the f's and y's have corresponding meanings to those in §8.

Thus

$$(D^2 + \Omega^2 + c'_1/p'_1)\,\phi'_1 = (l_1/p'_1 y'_{11})\,\Sigma f'_n y''_{n1} \quad . \quad . \quad (3)$$

or

$$(D^2 + \Omega^2 + c'_1/p'_1)\,\phi'_1 = T'_1$$

where T'_1 is a Fourier series of fundamental frequency that of the engine impulses.

Hence the frequency of the blade for the motion considered will be

$$(1/2\pi)\sqrt{(\Omega^2 + c'_1/p'_1)} \quad . \quad . \quad . \quad . \quad . \quad (4)$$

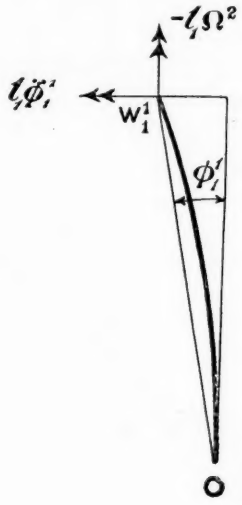


FIG. 7.

and resonance may occur when

$$(1/2\pi)\sqrt{(\Omega^2 + c'_1/p'_1)} = qnN/120 \quad . \quad . \quad . \quad (5)$$

10. Take next the case when the airscrew is close up to a short bearing so that the airscrew shaft at the point of attachment of the airscrew can adopt any slope. The gear wheel will be supposed negligible in comparison with the airscrew both as regards weight and moments of inertia.

The airscrew is regarded as being so close up to the nearest bearing that only angular deflection is appreciable.

To consider the motion as shown in Fig. 8, assuming that the shaft and the two conjugate blades of the airscrew are bent in one plane.

The applied periodic forces are neglected as they only affect the forced vibrations.

For the arms of P_1 in the plane of bending we have

$$c'_1\phi'_1 = -p_1\,(D^2 + \Omega^2)\,(\Phi_1 + \phi'_1) \quad . \quad . \quad . \quad (1)$$

or

$$[(D^2 + \Omega^2)/c'_1 + 1/p'_1]\,c'_1\phi'_1 = -(D^2 + \Omega^2)\,\Phi_1 \quad . \quad . \quad (2)$$

and

$$c'_1\phi'_2 = -p_1\,(D^2 + \Omega^2)\,(\Phi_1 + \phi'_2) \quad . \quad . \quad . \quad (3)$$

or

$$[(D^2 + \Omega^2)/c'_1 + 1/p'_1]\,c'_1\phi'_2 = -(D^2 + \Omega^2)\,\Phi_1 \quad . \quad . \quad (4)$$

The inertia couple bending the shaft at P_1 will be

$$c'_1(\phi'_1 + \phi'_2) \quad . \quad . \quad . \quad . \quad . \quad (5)$$

In the case of a four-bladed airscrew the other pair of blades will have no effect in the motion considered. Thus for the shaft at P_1

$$\Phi_1 = c'_1(\phi'_1 + \phi'_2)\phi_{11} \quad . \quad . \quad . \quad . \quad . \quad (6)$$

where ϕ_{11} is the slope due to unit couple.

From (2) and (4) we have for the blades a frequency

$$(1/2\pi)\sqrt{(\Omega^2 + c'_1/p'_1)} \quad . \quad . \quad . \quad . \quad . \quad (7)$$

From (2), (4) and (6) we find

$$[D^2 + \Omega^2 + c'_1/p'_1(1 + 2c'_1\phi_{11})]\Phi_1 = 0 \quad . \quad . \quad . \quad (8)$$

leading to a frequency for the system

$$(1/2\pi)\sqrt{[\Omega^2 + c'_1/p'_1(1 + 2c'_1\phi_{11})]} \quad . \quad . \quad . \quad (9)$$

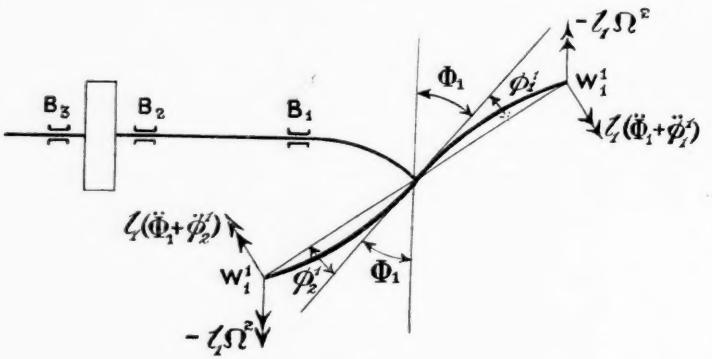


FIG. 8.

If there are three bearings, B_1, B_2, B_3, and $B_1B_2 = l_1$ and $B_2B_3 = l_2$,

$$\phi_{11} = l_1(3l_1 + 4l_2)/12EI(l_1 + l_2) \quad . \quad . \quad . \quad (10)$$

If there is no third bearing B_3

$$\phi_{11} = l_1/3EI \quad . \quad . \quad . \quad . \quad . \quad (11)$$

EI being the flexural rigidity of the shaft in the bending considered.

There will be resonance between the frequency (7) and the frequency of the firing impulses when

$$(1/2\pi)\sqrt{(\Omega^2 + c'_1/p'_1)} = qnN/120 \quad . \quad . \quad . \quad (12)$$

or

$$N = (60/\pi)\sqrt{[c'_1/p'_1(q^2n^2 - 4)]} \quad . \quad . \quad . \quad (13)$$

since

$$\Omega = \pi N/30$$

N is in r.p.m., n the number of cylinders and q any whole number.

Similarly, there will be resonance for the frequency (9) when

$$N = (60/\pi)\sqrt{[c'_1/p'_1(1 + 2c'_1\phi_{11})(q^2n^2 - 4)]} \quad . \quad . \quad (14)$$

The torsional oscillations have already been dealt with in §8.

Suppose next that the gear wheel is of appreciable inertia, and that B_2 is a short bearing so that the bending of the shaft to the left of B_2 affects the bending

of the shaft to the right. P_2 will be regarded as so close to the bearing B_2 that only angular deflection there is appreciable. If Φ_2 is the angular displacement of the gear wheel in the plane of bending there will be an inertia couple at P_2

$$-(p''_2 D^2 + p'_2 \Omega^2)\,\Phi_2 \quad . \quad . \quad . \quad . \quad (15)$$

where $p''_2 = I''_2/g$, I''_2 being the moment of inertia of P_2 about a line through its mass centre perpendicular to the plane of bending and $p'_2 = (I_2 - I_3)/g$ the difference divided by g between its polar moment of inertia and the moment of inertia about the line through its mass centre and in the plane of bending.

If gp_2 is the polar moment of inertia of the gear wheel

$$p'_2 = \tfrac{1}{2}p_2 \text{ approximately.}$$
$$p''_2 = \tfrac{1}{2}p_2 \text{ approximately.}$$

For the equations of motion we have

$$\Phi_1 = c'_1(\phi'_1 + \phi'_2)\,\phi_{11} - \tfrac{1}{2}p_2(D^2 + \Omega^2)\,\phi_{12}\Phi_2 \quad . \quad . \quad (16)$$
$$\Phi_2 = c'_1(\phi'_1 + \phi'_2)\,\phi_{12} - \tfrac{1}{2}p_2(D^2 + \Omega^2)\,\phi_{22}\Phi_2 \quad . \quad . \quad (17)$$

where ϕ_{11} = slope at P_1 due to unit couple at P_1.
ϕ_{22} = slope at P_2 due to unit couple at P_2.
ϕ_{12} = slope at P_2 due to unit couple at P_1.
= slope at P_1 due to unit couple at P_2.

From (16) and (17) we have

$$\Phi_2\phi_{11} - \Phi_1\phi_{12} = -\tfrac{1}{2}p_2(D^2 + \Omega^2)\,\Delta_{12}\Phi_2 \quad . \quad . \quad . \quad (18)$$

or

$$[\tfrac{1}{2}p_2(D^2 + \Omega^2)\,\Delta_{12} + \phi_{11}]\,\Phi_2 - \phi_{12}\Phi_1 = 0 \quad . \quad . \quad (19)$$

where

$$\Delta_{12} = \phi_{11}\phi_{12} - \phi_{12}^2$$

For the blades of the airscrew we have

$$[(D^2 + \Omega^2)/c'_1 + 1/p'_1]\,[c'_1\phi'_1 \text{ or } c'_1\phi'_2] = -(D^2 + \Omega^2)\,\Phi_1 \quad . \quad (20)$$

Using the equation (20), (17) becomes

$$[(D^2 + \Omega^2)/c'_1 + 1/p'_1]\,[\tfrac{1}{2}p_2(D^2 + \Omega^2)\,\phi_{22} + 1]\,\Phi_2 + 2(D^2 + \Omega^2)\,\phi_{12}\Phi_1 = 0 \quad . \quad . \quad . \quad . \quad (21)$$

From (19) and (21) we obtain

$$\{[(D^2 + \Omega^2)/c'_1 + 1/p'_1]\,[\tfrac{1}{2}p_2(D^2 + \Omega^2)\,\phi_{22} + 1] + 2(D^2 + \Omega^2)\,[\tfrac{1}{2}p_2(D^2 + \Omega^2)\,\Delta_{12} + \phi_{11}]\}\,[\Phi_1 \text{ or } \Phi_2] = 0 \quad . \quad (22)$$

Thus there will be two harmonics of frequencies $k_1/2\pi$, $k_2/2\pi$ where k_1^2, k_2^2 are roots of the equation in k^2.

$$r(k^2 - \Omega^2)^2 - s(k^2 - \Omega^2) + t = 0 \quad . \quad . \quad . \quad (23)$$

where $r = p'_1p_2(\phi_{22} + 2c'_1\Delta_{12})$
$s = 2p'_1 + c'_1(4p'_1\phi_{11} + p_2\phi_{22})$
and $t = 2c'_1$

For resonance we have

$$k^2 - \Omega^2 = \pi^2N^2(q^2n^2 - 4)/3600 \quad . \quad . \quad . \quad (24)$$

where N is r.p.m., n the number of cylinders and q any whole number.

Equation (23) in k^2 can thus be converted into one for N^2 to obtain the resonance speeds.

In addition, there will be the fore and aft frequency

$$(1/2\pi)\sqrt{(\Omega^2 + c'_1/p'_1)} \quad . \quad . \quad . \quad . \quad (25)$$

for each blade as previously found. The torsional motion will also be the same as previously considered.

PART II.

Investigation of the Principal Types of Engine

1. Fig. 9 is a diagrammatic outline of the main rotating parts of a single-throw crank radial engine with the airscrew coupled direct to the crankshaft. P_1 is the airscrew; W_2 the estimated concentrated load at the crankpin; W_3, W_4 balance loads; B_1, B_2, B_3 bearings.

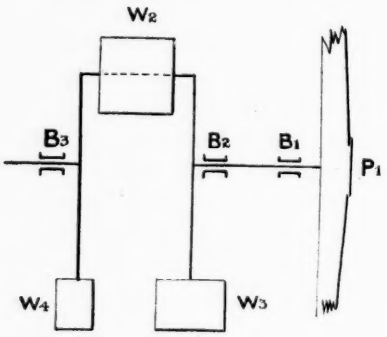


FIG. 9.

As a first approximation P_1 and the cranks are regarded as rigid.

We have already found (Part I., §2) that such a system has a frequency

$$(1/2\pi)\sqrt{[c_{12}(1/p_1 + 1/p_2)]} \quad . \quad . \quad . \quad . \quad (1)$$

where $p_1 = I_1/g$ and $p_2 = I_2/g$; I_1 being the moment of inertia of the airscrew about its axis of revolution and I_2 the moment of inertia of the crank and its loads (the load at the crankpin being regarded as concentrated) about the axis of the crankshaft; and c_{12} is the torsional stiffness of the shaft between P_1 and P_2.

For torsional resonance we have

$$(1/2\pi)\sqrt{[c_{12}(1/p_1 + 1/p_2)]} = qnN/120 \quad . \quad . \quad . \quad (2)$$

As a next approximation the blades of the airscrew are regarded as flexible, but the crank is still regarded as rigid.

The motion of such a system has already been investigated in Part I., §8.

For a two-bladed airscrew there are two frequencies, viz.:

$$(1/2\pi)\sqrt{(c_1/p_1)},\ (1/2\pi)\sqrt{[(1/2p_1 + 1/p_2)/(1/2c_1 + 1/c_{12})]} \quad . \quad (3)$$

the former referring to the blades only, where p_1 is $W_1 l_1^2/g$, W_1 being the equivalent tip load for each blade; c_1 is the equivalent flexural coefficient for each blade and p_2 and c_{12} are the same as for (2) above.

In addition each blade has a frequency

$$(1/2\pi)\sqrt{(\Omega^2 + c'_1/p'_1)} \quad . \quad . \quad . \quad . \quad . \quad (4)$$

for fore and aft oscillations.

2. We next take into consideration the flexibility of the crank webs as well as the flexibility of the blades of the airscrew.

The crank with pin is taken as one flexible arm and the balance cranks are taken as another separate flexible arm. We have then a system as shown diagrammatically in Fig. 10.

The system P_2 is replaced by two loads, W_2, W_3, at the ends of collinear flexible arms of lengths l_2, l_3.

Suppose in the first case the airscrew is two-bladed.

In order to obtain the frequencies of such a system we will neglect the applied torques and friction at the bearings as these only affect the forced vibrations.

Let θ_1 be the angular displacement of the normal line of the blades of the airscrew and θ_2 that of the normal line of the arms l_2, l_3.

For the blades of the airscrew we have, with the general notation

$$(D^2/c_1 + 1/p_1)\,[c_1\phi_1 \text{ or } c_1\phi_2] = -D^2\theta_1 \quad . \quad . \quad . \quad (1)$$

From which we have at one of the frequencies of the blades

$$(1/2\pi)\sqrt{(c_1/p_1)} \quad . \quad . \quad . \quad . \quad . \quad (2)$$

For the pair of blades we have by moments

$$c_1(\phi_1 + \phi_2) + c_{12}\theta_{12} = 0 \quad . \quad . \quad . \quad . \quad (3)$$

For the arms l_2, l_3, we have

$$(D^2/c_2 + 1/p_2)\,c_2\phi_5 = -D^2\theta_2 \quad . \quad . \quad . \quad . \quad (4)$$

$$(D^2/c_3 + 1/p_3)\,c_3\phi_6 = -D^2\theta_2 \quad . \quad . \quad . \quad . \quad (5)$$

and by moments

$$c_2\phi_5 + c_3\phi_6 - c_{12}\theta_{12} = 0 \quad . \quad . \quad . \quad . \quad (6)$$

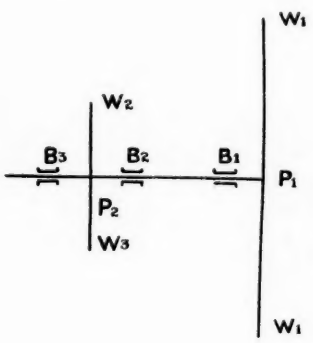


FIG. 10.

From (1) and (3) we obtain

$$2D^2\theta_1 - c_{12}(D^2/c_1 + 1/p_1)\,\theta_{12} = 0 \quad . \quad . \quad . \quad (7)$$

and from (4), (5) and (6)

$$D^2\left[(1/c_2 + 1/c_3)\,D^2 + 1/p_2 + 1/p_3\right]\theta_2 + c_{12}(D^2/c_2 + 1/p_2)(D^2/c_3 + 1/p_3)\,\theta_{12} = 0 \quad . \quad . \quad (8)$$

By (7) and (8) we find two frequencies $k_1/2\pi$, $k_2/2\pi$, where k_1^2, k_2^2, are roots of the equation in k^2.

$$rk^4 - sk^2 + t = 0 \quad . \quad . \quad . \quad . \quad . \quad (9)$$

where

$$r = (1/2c_1 + 1/c_{12})(1/c_2 + 1/c_3) + 1/c_2c_3$$

$$s = (1/2c_1 + 1/c_{12})(1/p_2 + 1/p_3) + (1/c_2)(1/2p_1 + 1/p_3) + (1/c_3)(1/2p_1 + 1/p_2)$$

and

$$t = (1/2p_1)(1/p_2 + 1/p_3) + 1/p_2p_3$$

In addition, each blade will h[ave a] frequency

$$(1 \quad .)^2 + c'_1/p'_1) \quad . \quad . \quad . \quad . \quad . \quad (10)$$

for fore and aft oscillations.

For a four-bladed airscrew [with]

(1) The blade frequency

$$(\quad \sqrt{(c_1/p_1)} \quad . \quad . \quad . \quad . \quad . \quad (11)$$

(2) Two frequencies $k_1/2\pi$, [$k_2/2\pi$], where k_1^2, k_2^2 are roots of the equation in k^2.

$$rk^4 - \quad - o \quad . \quad . \quad . \quad . \quad (12)$$

where

$$r = (1/4c_1 + 1/c_{12})(1/c_2 + 1/c_3) + 1/c_2c_3$$

$$s = (1/4c_1 + 1/c_{12})(1/p_2 + 1/p_3) + (1/c_2)(1/4p_1 + 1/\quad) + (1/c_3)(1/4p_1 + 1/p_2)$$

and

$$t = (1/4p_1)(1/p_2 + 1/p_3) + 1/p_2p_3$$

In addition, each blade will have a frequency

$$(1/2\pi)\sqrt{(\Omega^2 + c'_1/p'_1)} \quad . \quad . \quad . \quad . \quad (13)$$

for fore and aft oscillations.

Similarly, the arms l_2, l_3, will have frequencies

$$(1/2\pi)\sqrt{(\Omega^2 + c'_2/p'_2)},\ (1/2\pi)\sqrt{(\Omega^2 + c'_3/p'_3)} \quad . \quad . \quad (14)$$

for fore and aft oscillations.

3. As a final refinement we regard each balance crank as an independent arm, as in Fig. 9.

The system P_2 is now replaced by three flexible arms l_2, l_3, l_4, each carrying a load at its extremity. For the airscrew blades (supposed two)

$$(D^2/c_1 + 1/p_1)[c_1\phi_1 \text{ or } c_1\phi_2] = -D^2\theta_1 \quad . \quad . \quad . \quad (1)$$

and for the pair by moments

$$c_1(\phi_1 + \phi_2) + c_{12}\theta_{12} = o \quad . \quad . \quad . \quad . \quad (2)$$

For the arms l_2, l_3, l_4, we have

$$(D^2/c_2 + 1/p_2)\, c_2\phi_5 = -D^2\theta_2 \quad . \quad . \quad . \quad . \quad (3)$$

$$(D^2/c_3 + 1/p_3)\, c_3\phi_6 = -D^2\theta_2 \quad . \quad . \quad . \quad . \quad (4)$$

$$(D^2/c_4 + 1/p_4)\, c_4\phi_7 = -D^2\theta_2 \quad . \quad . \quad . \quad . \quad (5)$$

and by moments

$$c_2\phi_5 + c_3\phi_6 + c_4\phi_7 - c_{12}\theta_{12} = o \quad . \quad . \quad . \quad . \quad (6)$$

From (1) we obtain the blade frequency

$$(1/2\pi)\sqrt{(c_1/p_1)} \quad . \quad . \quad . \quad . \quad . \quad (7)$$

From (1) and (2) we have

$$-2D^2\theta_1 + c_{12}(D^2/c_1 + 1/p_1)\,\theta_{12} = o \quad . \quad . \quad . \quad (8)$$

and from (3), (4), (5) and (6) we have

$$D^2[(D^2/c_2 + 1/p_2)(D^2/c_3 + 1/p_3) + (D^2/c_2 + 1/p_2)(D^2/c_4 + 1/p_4) + (D^2/c_3 + 1/p_3)(D^2/c_4 + 1/p_4)]\,\theta_2 + c_{12}(D^2/c_2 + 1/p_2)(D^2/c_3 + 1/p_3)(D^2/c_4 + 1/p_4)\,\theta_{12} = o \quad . \quad (9)$$

(8) can be written

$$-D^2\theta_2 + c_{12}[(1/2c_1 + 1/c_{12})D^2 + 1/2p_1]\,\theta_{12} = o \quad . \quad . \quad (10)$$

From (9) and (10) we obtain three frequencies, $k_1/2\pi$, $k_2/2\pi$, $k_3/2\pi$, where k_1^2, k_2^2, k_3^2, are roots of the equation in k^2.

$$rk^6 - sk^4 + tk^2 - u = o \quad . \quad . \quad . \quad . \quad (11)$$

where

$$r = (1/2c_1 + 1/c_{12})(1/c_2c_3 + 1/c_2c_4 + 1/c_3c_4) + 1/c_2c_3c_4$$

$$s = (1/2c_1 + 1/c_{12})[(1/c_2)(1/p_3 + 1/p_4) + (1/c_3)(1/p_2 + 1/p_4) + (1/c_4)(1/p_2 + 1/p_3)] + (1/c_2c_3)(1/2p_1 + 1/p_4) + (1/c_2c_4)(1/2p_1 + 1/p_3) + (1/c_3c_4)(1/2p_1 + 1/p_2)$$

$$t = (1/2c_1 + 1/c_{12})(1/p_2p_3 + 1/p_2p_4 + 1/p_3p_4) + (1/c_2)[(1/2p_1)(1/p_3 + 1/p_4) + 1/p_3p_4] + (1/c_3)[(1/2p_1)(1/p_2 + 1/p_4) + 1/p_2p_4] + (1/c_4)[(1/2p_1)(1/p_2 + 1/p_3) + 1/p_2p_3]$$

$$u = (1/2p_1)(1/p_2p_3 + 1/p_2p_4 + 1/p_3p_4) + 1/p_2p_3p_4$$

In addition there will be for each blade a frequency

$$(1/2\pi)\sqrt{(\Omega^2 + c'_1/p'_1)}$$

for fore and aft oscillations and similarly frequencies

$$(1/2\pi)\sqrt{(\Omega^2 + c'_2/p'_2)},\ (1/2\pi)\sqrt{(\Omega^2 + c'_3/p'_3)},\ (1/2\pi)\sqrt{(\Omega^2 + c'_4/p'_4)},$$

for the fore and aft oscillations of the arms l_2, l_3, l_4, respectively.

In the case of a four-bladed airscrew we write $2c_1$ for c_1 and $2p_1$ for p_1 in (11).

4. Consider next a two-throw crankshaft engine as depicted diagrammatically in Fig. 11.

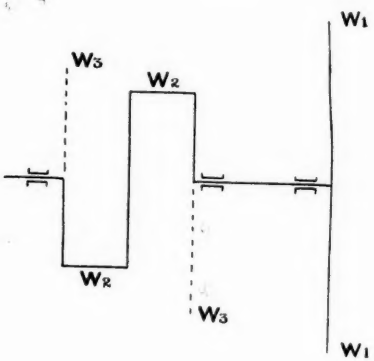


FIG. 11.

We assume that the throws are equal in all respects, that the concentrated loads at the crankpins are equal and that in the first place there are no balance loads.

We have for the blades of the airscrew

$$(D^2/c_1 + 1/p_1)\ [c_1\phi_1 \text{ or } c_1\phi_2] = -D^2\theta_1 \quad . \quad . \quad . \quad (1)$$

and by moments for the two blades

$$c_1(\phi_1 + \phi_2) + c_{12}\theta_{12} = 0 \quad . \quad . \quad . \quad . \quad (2)$$

Similarly for the two cranks

$$(D^2/c_2 + 1/p_2)\ [c_2\phi_5 \text{ or } c_2\phi_6] = -D^2\theta_2 \quad . \quad . \quad . \quad (3)$$

and by moments for the two cranks

$$c_2(\phi_5 + \phi_6) - c_{12}\theta_{12} = 0 \quad . \quad . \quad . \quad . \quad (4)$$

From (1) we have as one of the frequencies of the blades

$$(1/2\pi)\sqrt{(c_1/p_1)} \quad . \quad . \quad . \quad . \quad . \quad (5)$$

Similarly from (3) we have as one of the frequencies of the cranks

$$(1/2\pi)\sqrt{(c_2/p_2)} \quad . \quad . \quad . \quad . \quad . \quad (6)$$

From (1) and (2) we obtain

$$-2D^2\theta_1 + c_{12}(D^2/c_1 + 1/p_1)\theta_{12} = 0 \quad . \quad . \quad . \quad (7)$$

and from (3) and (4) we obtain

$$-2D^2\theta_2 - c_{12}(D^2/c_2 + 1/p_2)\theta_{12} = 0 \quad . \quad . \quad . \quad (8)$$

From (7) and (8) we find

$$[(1/2c_1 + 1/2c_2 + 1/c_{12})D^2 + 1/2p_1 + 1/2p_2]\theta_{12} = 0 \quad . \quad . \quad (9)$$

and thus we obtain the frequency

$$(1/2\pi)\sqrt{[(1/2p_1 + 1/2p_2)/(1/2c_1 + 1/2c_2 + 1/c_{12})]} \quad . \quad . \quad (10)$$

In addition the blades of the airscrew will have a frequency

$$(1/2\pi)\sqrt{(\Omega^2 + c'_1/p'_1)} \quad . \quad . \quad . \quad . \quad (11)$$

and the cranks a frequency

$$(1/2\pi)\sqrt{(\Omega^2 + c'_2/p'_2)} \quad . \quad . \quad . \quad . \quad (12)$$

for fore and aft oscillations.

If the airscrew is four-bladed we write $2c_1$ for c_1 and $2p_1$ for p_1 in (10).

5. If now there are two equal balance arms W_3, as shown dotted in Fig. 11, we have:

For the airscrew blades

$$(D^2/c_1 + 1/p_1)\,[c_1\phi_1 \text{ or } c_1\phi_2] = -D^2\theta_1 \quad . \quad . \quad . \quad (1)$$

and by moments

$$c_1(\phi_1 + \phi_2) + c_{12}\theta_{12} = 0 \quad . \quad . \quad . \quad . \quad (2)$$

For the two cranks

$$(D^2/c_2 + 1/p_2)\,[c_2\phi_5 \text{ or } c_2\phi_6] = -D^2\theta_2 \quad . \quad . \quad . \quad (3)$$

For the balance cranks

$$(D^2/c_3 + 1/p_3)\,[c_3\phi_7 \text{ or } c_3\phi_8] = -D^2\theta_2 \quad . \quad . \quad . \quad (4)$$

By moments for the cranks and balance cranks

$$c_2(\phi_5 + \phi_6) + c_3(\phi_7 + \phi_8) - c_{12}\theta_{12} = 0 \quad . \quad . \quad . \quad (5)$$

From (1) we have as one of the frequencies of the blades

$$(1/2\pi)\sqrt{(c_1/p_1)} \quad . \quad . \quad . \quad . \quad . \quad (6)$$

From (3) one of the frequencies of the cranks is

$$(1/2\pi)\sqrt{(c_2/p_2)} \quad . \quad . \quad . \quad . \quad . \quad (7)$$

and from (4) one of the frequencies of the balance cranks is

$$(1/2\pi)\sqrt{(c_3/p_3)} \quad . \quad . \quad . \quad . \quad . \quad (8)$$

From (1) and (2) we have

$$-2D^2\theta_1 + c_{12}(D^2/c_1 + 1/p_1)\theta_{12} = 0 \quad . \quad . \quad . \quad (9)$$

and from (3), (4) and (5)

$$2D^2[(1/c_2 + 1/c_3)D^2 + 1/p_2 + 1/p_3]\theta_2 + c_{12}(D^2/c_2 + 1/p_2)(D^2/c_3 + 1/p_3)\theta_{12} = 0 \quad . \quad . \quad (10)$$

Thus from (9) and (10) we obtain two frequencies $k_1/2\pi$, $k_2/2\pi$, where k_1^2, k_2^2, are the roots of the equation in k^2.

$$rk^4 - sk^2 + t = 0 \quad . \quad . \quad . \quad . \quad . \quad (11)$$

where

$$r = 2(1/2c_1 + 1/c_{12})(1/c_2 + 1/c_3) + 1/c_2c_3$$

$$s = 2(1/2c_1 + 1/c_{12})(1/p_2 + 1/p_3) + (1/c_2 + 1/c_3)(1/p_1) + 1/p_2c_3 + 1/p_3c_2$$

$$t = (1/p_1)(1/p_2 + 1/p_3) + 1/p_2p_3$$

In addition each blade will have the frequency

$$(1/2\pi)\sqrt{(\Omega^2 + c'_1/p'_1)} \quad . \quad . \quad . \quad . \quad (12)$$

each crank the frequency

$$(1/2\pi)\sqrt{(\Omega^2 + c'_2/p'_2)} \quad . \quad . \quad . \quad . \quad (13)$$

and each balance crank the frequency

$$(1/2\pi)\sqrt{(\Omega^2 + c'_3/p'_3)} \quad . \quad . \quad . \quad . \quad (14)$$

for fore and aft oscillations.

In the case of a four-bladed airscrew we write $2c_1$ for c_1 and $2p_1$ for p_1 in (11).

6. Take next a four-throw crankshaft as shown in Fig. 12.

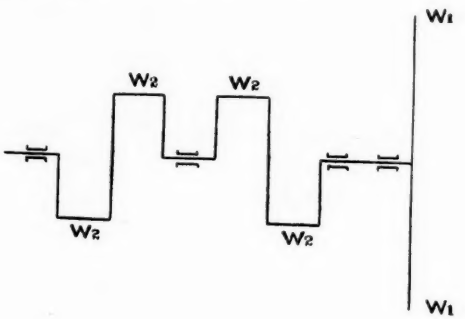


FIG. 12.

We assume that the throws are equal in all respects, that the concentrated loads at the pins are all equal and that there are no balance loads.

Let θ_1 be the angular displacement of the normal line of the airscrew blades, θ_2 the angular displacement of the normal line of the pair of cranks adjacent to the airscrew and θ_3 that for the remaining pair of cranks.

For the airscrew blades we have

$$(D^2/c_1 + 1/p_1)\,[c_1\phi_1 \text{ or } c_1\phi_2] = -D^2\theta_1 \quad . \quad . \quad . \quad (1)$$

and by moments

$$c_1(\phi_1 + \phi_2) + c_{12}\theta_{12} = 0 \quad . \quad . \quad . \quad . \quad (2)$$

For the pair of cranks adjacent to the airscrew we have

$$(D^2/c_2 + 1/p_2)\,[c_2\phi_5 \text{ or } c_2\phi_6] = -D^2\theta_2 \quad . \quad . \quad . \quad (3)$$

and by moments

$$c_2(\phi_5 + \phi_6) + c_{22}\theta_{23} - c_{12}\theta_{12} = 0 \quad . \quad . \quad . \quad . \quad (4)$$

where c_{22} refers to the journal between the two pairs of cranks and

$$\theta_{23} = \theta_3 - \theta_2$$

Similarly for the other pair of cranks

$$(D^2/c_2 + 1/p_2)\,[c_2\phi_7 \text{ or } c_2\phi_8] = -D^2\theta_2 \quad . \quad . \quad . \quad (5)$$

and by moments

$$c_2(\phi_7 + \phi_8) - c_{22}\theta_{23} = 0 \quad . \quad . \quad . \quad . \quad (6)$$

From (1) we have as one of the frequencies of the blades

$$(1/2\pi)\sqrt{(c_1/p_1)} \quad . \quad . \quad . \quad . \quad . \quad (7)$$

and from (3) and (5) one of the frequencies of the cranks

$$(1/2\pi)\sqrt{(c_2/p_2)} \quad . \quad . \quad . \quad . \quad . \quad (8)$$

From (1) and (2) we obtain

$$-2D^2\theta_1 + (D^2/c_1 + 1/p_1)\,c_{12}\theta_{12} = 0 \quad . \quad . \quad . \quad (9)$$

From (3) and (4)

$$-2D^2\theta_2 + (D^2/c_2 + 1/p_2)(c_{22}\theta_{23} - c_{12}\theta_{12}) = 0 \quad . \quad . \quad (10)$$

and from (5) and (6)

$$-2D^2\theta_3 - (D^2/c_2 + 1/p_2)\, c_{22}\theta_{23} = 0 \quad . \quad . \quad . \quad (11)$$

From (9) and (10) we have

$$\{(2/c_{12} + 1/c_1 + 1/c_2)\, D^2 + 1/p_1 + 1/p_2]\, c_{12}\theta_{12} - (D^2/c_2 + 1/p_2)\, c_{22}\theta_{23} = 0 \quad (12)$$

and from (10) and (11)

$$-(D^2/c_2 + 1/p_2)\, c_{12}\theta_{12} + 2\,[(1/c_{22} + 1/c_2)\, D^2 + 1/p_2]\, c_{22}\theta_{23} = 0 \quad . \quad (13)$$

Thus we obtain two frequencies $k_1/2\pi$, $k_2/2\pi$, where k_1^2, k_2^2, are the roots of the equation in k^2.

$$rk^4 - sk^2 + t = 0 \quad . \quad . \quad . \quad . \quad . \quad (14)$$

where

$$r = 4\,(1/2c_1 + 1/c_{12})\,(1/c_{22} + 1/c_2) + 2/c_{22}c_2 + 1/c_2^2$$

$$s = 4\,(1/2c_1 + 1/c_{12})\,(1/p_2) + 2\,(1/c_{22} + 1/c_2)\,(1/p_1 + 1/p_2)$$

$$t = (2/p_2)\,(1/p_1 + 1/2p_2)$$

In addition there are the frequencies

$$(1/2\pi)\sqrt{(\Omega^2 + c'_1/p'_1)},\ (1/2\pi)\sqrt{(\Omega^2 + c'_2/p'_2)} \quad . \quad . \quad (15)$$

for the blades and cranks respectively for fore and aft oscillations.

For four-bladed airscrews we write $2c_1$ for c_1 and $2p_1$ for p_1 in (14).

7. Consider next a four-throw crankshaft, there being three equal intermediate journals. All the cranks are taken as equal in all respects. The engine is geared and the gear wheel driving the airscrew gear is supposed of small mass and inertia. Fig. 13 is a diagrammatic representation of such a system.

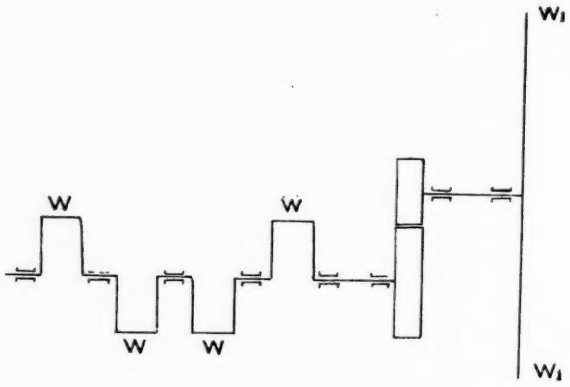


FIG. 13.

In this case the system is split up into two parts. There is the system comprising the airscrew, the airscrew shaft and the gear wheel driving the latter. The other system is the crankshaft and its gear wheel.

We will first ignore the airscrew system.

Let θ_1, θ_2, θ_3, θ_4, be the angular displacements of the normal lines of the cranks (numbering from the airscrew end).

Let c_2 and p_2 refer to each crank and c_{22} to each journal.

We have for the cranks

$$(D^2/c_2 + 1/p_2)\, c_2\phi_r = -D^2\theta_r \quad . \quad . \quad . \quad . \quad (1)$$

where $r = 1, 2, 3$ or 4, or

$$\delta_2 c_2\phi_r = -D^2\theta_r \quad . \quad . \quad . \quad . \quad . \quad (2)$$

where

$$\delta_2 = D^2/c_2 + 1/p_2$$

Also by moments

$$c_2\phi_1 + c_{22}\theta_{12} = 0 \quad . \quad . \quad . \quad . \quad . \quad (3)$$
$$c_2\phi_2 + c_{22}(\theta_{23} - \theta_{12}) = 0 \quad . \quad . \quad . \quad . \quad (4)$$
$$c_2\phi_3 + c_{22}(\theta_{34} - \theta_{23}) = 0 \quad . \quad . \quad . \quad . \quad (5)$$

and

$$c_2\phi_4 - c_{22}\theta_{34} = 0 \quad . \quad . \quad . \quad . \quad . \quad (6)$$

From (1) we find

$$\delta_2 c_2(\phi_2 - \phi_1) = -D^2\theta_{12} \quad . \quad . \quad . \quad . \quad (7)$$

or

$$\delta_2 c_2\phi_{12} = -D^2\theta_{12} \quad . \quad . \quad . \quad . \quad . \quad (8)$$

where

$$\phi_{12} = \phi_2 - \phi_1$$

Similarly

$$\delta_2 c_2\phi_{23} = -D^2\theta_{23} \quad . \quad . \quad . \quad . \quad . \quad (9)$$

and

$$\delta_2 c_2\phi_{34} = -D^2\theta_{34} \quad . \quad . \quad . \quad . \quad . \quad (10)$$

From (3), (4), (5) and (6) we find

$$\delta_2 c_2\phi_{12} + \delta_2 c_{22}(\theta_{23} - 2\theta_{12}) = 0 \quad . \quad . \quad . \quad . \quad (11)$$
$$\delta_2 c_2\phi_{23} + \delta_2 c_{22}(\theta_{12} + \theta_{34} - 2\theta_{23}) = 0 \quad . \quad . \quad . \quad (12)$$

and

$$\delta_2 c_2\phi_{34} + \delta_2 c_{22}(\theta_{23} - 2\theta_{34}) = 0 \quad . \quad . \quad . \quad . \quad (13)$$

By pairing (8), (9) and (10) with (11), (12) and (13) respectively we obtain

$$\delta_3\theta_{12} - \delta_2\theta_{23} = 0 \quad . \quad . \quad . \quad . \quad . \quad (14)$$
$$-\delta_2\theta_{12} + \delta_3\theta_{23} - \delta_2\theta_{34} = 0 \quad . \quad . \quad . \quad . \quad (15)$$

and

$$-\delta_2\theta_{23} + \delta_3\theta_{34} = 0 \quad . \quad . \quad . \quad . \quad . \quad (16)$$

where

$$\delta_3 = D^2/c_{22} + 2\delta_2$$

From (14), (15) and (16) we find there are three frequencies, $k_1/2\pi$, $k_2/2\pi$, $k_3/2\pi$, where k_1^2, k_2^2, k_3^2, are the roots of the equation in k^2.

$$\Delta_4 = K^6 - 2K^2 = 0 \quad . \quad . \quad . \quad . \quad . \quad (17)$$

where

$$K^2 = [(1/c_{22} + 2/c_2)\, k^2 - 2/p_2]/(k^2/c_2 - 1/p_2)$$

Thus we obtain the frequencies

$$(1/2\pi)\sqrt{[(2 - \sqrt{2})/p_2\,\{\,1/c_{22} + (2 - \sqrt{2})/c_2\,\}\,]}$$

or

$$(1/2\pi)\sqrt{[(.586)/p_2\,(1/c_{22} + .586/c_2)]} \quad . \quad . \quad . \quad (18)$$
$$(1/2\pi)\sqrt{[2/p_2\,(1/c_{22} + 2/c_2)]} \quad . \quad . \quad . \quad . \quad (19)$$

and

$$(1/2\pi)\sqrt{[(2 + \sqrt{2})/p_2\,\{\,1/c_{22} + (2 + \sqrt{2})/c_2\,\}\,]}$$

or

$$(1/2\pi)\sqrt{[(3.414)/p_2\,(1/c_{22} + 3.414/c_2)]} \quad . \quad . \quad . \quad (20)$$

In addition the cranks have the frequency

$$(1/2\pi)\sqrt{(\Omega^2 + c'_2/p'_2)}$$

for fore and aft oscillations.

8. Take next the case of a six-throw crankshaft under the same conditions as in the preceding example.

With the same notation we have for the six cranks

$$\delta_2 c_2 \phi_r = -D^2 \theta_r \quad . \quad . \quad . \quad . \quad . \quad (1)$$

where $r = 1, 2, 3, 4, 5$ or 6, and by moments

$$c_2\phi_1 + c_{22}\theta_{12} = 0 \quad . \quad . \quad . \quad . \quad . \quad . \quad (2)$$
$$c_2\phi_2 + c_{22}(\theta_{23} - \theta_{12}) = 0 \quad . \quad . \quad . \quad . \quad (3)$$
$$c_2\phi_3 + c_{22}(\theta_{34} - \theta_{23}) = 0 \quad . \quad . \quad . \quad . \quad (4)$$
$$c_2\phi_4 + c_{22}(\theta_{45} - \theta_{34}) = 0 \quad . \quad . \quad . \quad . \quad (5)$$
$$c_2\phi_5 + c_{22}(\theta_{56} - \theta_{15}) = 0 \quad . \quad . \quad . \quad . \quad (6)$$

and

$$c_2\phi_6 - c_{22}\theta_{56} = 0 \quad . \quad . \quad . \quad . \quad . \quad . \quad (7)$$

The corresponding equation in K^2 is found to be

$$\Delta_6 = K^{10} - 4K^6 + 3K^2 = 0 \quad . \quad . \quad . \quad . \quad (8)$$

From which we obtain the five frequencies

$$(1/2\pi)\sqrt{[(2-\sqrt{3})/p_2\{1/c_{22} + (2-\sqrt{3})/c_2\}]}$$

or

$$(1/2\pi)\sqrt{[.268/p_2(1/c_{22} + .268/c_2)]} \quad . \quad . \quad . \quad (9)$$
$$(1/2\pi)\sqrt{[1/p_2(1/c_{22} + 1/c_2)]} \quad . \quad . \quad . \quad . \quad (10)$$
$$(1/2\pi)\sqrt{[2/p_2(1/c_{22} + 2/c_2)]} \quad . \quad . \quad . \quad . \quad (11)$$
$$(1/2\pi)\sqrt{[3/p_2(1/c_{22} + 3/c_2)]} \quad . \quad . \quad . \quad . \quad (12)$$

and

$$(1/2\pi)\sqrt{[(2+\sqrt{3})/p_2\{1/c_{22} + (2+\sqrt{3})/c_2\}]}$$

or

$$(1/2\pi)\sqrt{[3.732/p_2(1/c_{22} + 3.732/c_2)]} \quad . \quad . \quad . \quad (13)$$

In addition the cranks have the frequency

$$(1/2\pi)\sqrt{(\Omega^2 + c'_2/p'_2)}$$

for fore and aft oscillations.

9. For an eight-throw crankshaft under the same conditions we find the corresponding equation in K^2 is

$$\Delta_8 = K^{14} - 6K^{10} + 10K^6 - 4K^2 = 0 \quad . \quad . \quad . \quad (1)$$

From which we obtain the seven frequencies

$$(1/2\pi)\sqrt{[\{2-\sqrt{(2+\sqrt{2})}\}/p_2\{1/c_{22} + [2-\sqrt{(2+\sqrt{2})}]/c_2\}]}$$

or

$$(1/2\pi)\sqrt{[.152/p_2(1/c_{22} + .152/c_2)]} \quad . \quad . \quad . \quad (2)$$
$$(1/2\pi)\sqrt{[(2-\sqrt{2})/p_2\{1/c_{22} + (2-\sqrt{2})/c_2\}]}$$

or

$$(1/2\pi)\sqrt{[.586/p_2(1/c_{22} + .586/c_2)]} \quad . \quad . \quad . \quad (3)$$
$$(1/2\pi)\sqrt{[\{2-\sqrt{(2-\sqrt{2})}\}/p_2\{1/c_{22} + [2-\sqrt{(2-\sqrt{2})}]/c_2\}]}$$

or

$$(1/2\pi)\sqrt{[1.234/p_2(1/c_{22} + 1.235/c_2)]} \quad . \quad . \quad . \quad (4)$$
$$(1/2\pi)\sqrt{\{2/p_2(1/c_{22} + 2/c_2)\}} \quad . \quad . \quad . \quad . \quad (5)$$
$$(1/2\pi)\sqrt{[\{2+\sqrt{(2-\sqrt{2})}\}/p_2\{1/c_{22} + [2+\sqrt{(2-\sqrt{2})}]/c_2\}]}$$

or

$$(1/2\pi)\sqrt{[2.766/p_2(1/c_{22} + 2.765/c_2)]} \quad . \quad . \quad . \quad (6)$$
$$(1/2\pi)\sqrt{[(2+\sqrt{2})/p_2\{1/c_{22} + (2+\sqrt{2})/c_2\}]}$$

or

$$(1/2\pi)\sqrt{[3.414/p_2(1/c_{22} + 3.414/c_2)]} \quad . \quad . \quad . \quad (7)$$
$$(1/2\pi)\sqrt{[\{2+\sqrt{(2+\sqrt{2})}\}/p_2\{1/c_{22} + [2+\sqrt{(2+\sqrt{2})}]/c_2\}]}$$

or

$$(1/2\pi)\sqrt{[3.848/p_2\,(1/c_{22} + 3.848/c_2)]} \quad . \quad . \quad . \quad (8)$$

In addition the cranks have the frequency

$$(1/2\pi)\sqrt{(\Omega^2 + c'_2/p'_2)}$$

for fore and aft oscillations.

10. We find generally

$$\Delta_n = K^2\Delta_{n-1} - \Delta_{n-2} \quad . \quad . \quad . \quad . \quad . \quad (1)$$

Thus, Δ_n is the coefficient of x^{n-1} in the expansion of

$$1/(1 - K^2x + x^2) \quad . \quad . \quad . \quad . \quad . \quad (2)$$

in ascending power of x.

Actually

$$\Delta_n = K^{2n-2} - (n-2)\,K^{2n-6} + \tfrac{1}{2}(n-3)(n-4)\,K^{2n-10} - \{(n-4)(n-5)(n-6)/(1\cdot 2\cdot 3)\}\,K^{2n-14} + \ldots \quad . \quad . \quad (3)$$

The last term in the expression on the right hand side of (3) will be a multiple of K^2 when n is even and ± 1 when n is odd.

Let the roots in K^2 of

$$\Delta_n = 0 \quad . \quad . \quad . \quad . \quad . \quad . \quad (4)$$

be $\lambda_1, \lambda_2, \lambda_3, \lambda_4, \ldots \lambda_{n-1}$.

Then the corresponding k's are obtained from the relations

$$(1/c_{22} + 2/c_2)\,k^2 - 2/p_2 = \lambda_r\,(k^2/c_2 - 1/p_2) \quad . \quad . \quad (5)$$

or

$$k^2 = (2 - \lambda_r)/p_2\,[1/c_{22} + (2 - \lambda_r)/c_2] \quad . \quad . \quad . \quad (6)$$

From which we obtain the frequency

$$(1/2\pi)\sqrt{[(2 - \lambda_r)/p_2\,\{1/c_{22} + (2 - \lambda_r)/c_2\}]} \quad . \quad . \quad (7)$$

The values for $\sqrt{(2 - \lambda_r)}$ have been calculated in the cases

$$\Delta_n = 0$$

when $n = 1, 2, 3, \ldots$ or 10, and are given in Table I.

It can be shown that

$$\Delta_{2n} = \Delta_n\,(\Delta_{n+1} - \Delta_{n-1}) \quad . \quad . \quad . \quad . \quad (8)$$

so that a system of $2n$ equal cranks will have all the frequencies of a system of n of the same cranks.

Further, if $\lambda_1, \lambda_2, \lambda_3, \ldots \lambda_{2n-1}$ are the roots in K^2 of the equation

$$\Delta_{2n} = 0 \quad . \quad . \quad . \quad . \quad . \quad . \quad (9)$$

the values

$$\sqrt{(2 - \lambda_1)},\ \sqrt{(2 - \lambda_3)},\ \sqrt{(2 - \lambda_5)},\ \ldots\ \sqrt{(2 - \lambda_{2n-1})}$$

arise from

$$\Delta_{n+1} - \Delta_{n-1} = 0 \quad . \quad . \quad . \quad . \quad . \quad (10)$$

and

$$\sqrt{(2 - \lambda_2)},\ \sqrt{(2 - \lambda_1)},\ \sqrt{(2 - \lambda_6)},\ \ldots\ \sqrt{(2 - \lambda_{2n-2})}$$

arise from

$$\Delta_n = 0 \quad . \quad . \quad . \quad . \quad . \quad . \quad (11)$$

Similarly it can be shown that

$$\Delta_{2n+1} = \Delta^2_{n+1} - \Delta^2_n \quad . \quad . \quad . \quad . \quad (12)$$

and that if $\lambda_1, \lambda_2, \lambda_3, \ldots \lambda_{2n}$ are the roots in K^2 of the equation

$$\Delta_{2n+1} = 0 \quad . \quad . \quad . \quad . \quad . \quad (13)$$

the values

$$\sqrt{(2 - \lambda_1)},\ \sqrt{(2 - \lambda_3)},\ \sqrt{(2 - \lambda_5)},\ \ldots\ \sqrt{(2 - \lambda_{2n-1})}$$

arise from

$$\Delta_{n+1} - \Delta_n = 0 \quad . \quad . \quad . \quad . \quad . \quad (14)$$

TABLE I.

Values of $\sqrt{(2 - \lambda_r)}$ where $\lambda_1, \lambda_2, \lambda_3, \ldots \lambda_{n-1}$ are the roots in K^2 of the equation $\Delta_n = 0$.

n	2	3	4	5	6	7	8	9	10	APPROXN.
$\Delta_2 = K^2$	1.414	1.000	0.766	0.618	0.518	0.445	0.390	0.347	0.313	$3/n$
$\Delta_3 = K^4 - 1$	1.732	1.414	1.176	1.000	0.868	0.766	0.684	0.618		$7/(n+1)$
$\Delta_4 = K^6 - 2K^2$	1.848	1.618	1.414	1.247	1.111	1.000	0.908			$11/(n+2)$
$\Delta_5 = K^8 - 3K^4 + 1$	1.902	1.732	1.564	1.414	1.286	1.176				$15/(n+3)$
$\Delta_6 = K^{10} - 4K^6 + 3K^2$	1.932	1.802	1.663	1.532	1.414					$19/(n+4)$
$\Delta_7 = K^{12} - 5K^8 + 6K^4 - 1$	1.950	1.848	1.732	1.618						$23/(n+5)$
$\Delta_8 = K^{14} - 6K^{10} + 10K^6 - 4K^2$	1.962	1.880	1.782							$27/(n+6)$
$\Delta_9 = K^{16} - 7K^{12} + 15K^8 - 10K^4 + 1$	1.970	1.902								$31/(n+7)$
$\Delta_{10} = K^{18} - 8K^{14} + 21K^{10} - 20K^6 + 5K^2$	1.975									$35/(n+8)$

and

$$\sqrt{(2-\lambda_2)},\ \sqrt{(2-\lambda_4)},\ \ldots\ \sqrt{(2-\lambda_{2n})}$$

arise from

$$\Delta_{n+1} + \Delta_n = 0 \qquad . \qquad . \qquad . \qquad . \qquad . \qquad (15)$$

11. From the values of $\sqrt{(2-\lambda_r)}$ in Table I. we observe that as the number of cranks increases

$$\sqrt{(2-\lambda_r)} \longrightarrow 0$$
$$\text{as } n \longrightarrow \infty$$

where λ_r is the highest positive root of the equation in K^2

$$\Delta_n = 0$$

Also

$$\sqrt{(2-\lambda_r)} \longrightarrow 2$$
$$\text{as } n \longrightarrow \infty$$

where λ_r is numerically the greatest negative root.

Thus the greatest frequency possible is

$$(1/2\pi)\sqrt{[4/p_2\,(1/c_{22} + 4/c_2)]}$$

the lowest being zero.

The values in Table I. have remarkable approximations.

For example, if n be the number of cranks, the lowest value of $\sqrt{(2-\lambda_r)}$ is very approximately

$$3/n$$

This approximation is within 4 per cent., except in the case of two cranks when it is within 6.5 per cent. of the true value. The rule will be near enough for practice allowing a margin of 5 per cent.

Thus very approximately the fundamental frequency of torsional oscillations in the case of n cranks is

$$1/2\pi\sqrt{[p_2\,(n^2/9c_{22} + 1/c_2)]}$$

Generally $1/c_2$ will be small compared with $1/c_{22}$, so that with throws exceeding four the flexibility of the cranks will not usually be important.

For example, in a geared engine with a six-throw crankshaft let the dimensions of the journals be

$$2'' \times 1\tfrac{3}{4}'' \text{ (dia.)}$$

and of the crankwebs

$$3'' \times 2\tfrac{1}{2}'' \times 1''$$

Then

$$c_{22} = 5{\cdot}52 \times 10^6$$

and remembering there are two webs to one crank

$$c_2 = 78{\cdot}13 \times 10^6$$

Thus

$$n^2/9c_{22} + 1/c_2 = (4/5{\cdot}52 + 1/78{\cdot}13)\,10^{-6}$$

The flexibility of the cranks will decrease the fundamental frequency in the ratio

$$\sqrt{(4/5{\cdot}52)}/\sqrt{(4/5{\cdot}52 + 1/78{\cdot}13)}$$

i.e., approximately 155/156, or a decrease of less than 1 per cent.

If the estimated crankpin load is 24/3 lbs.

$$p_2 = (24/3) \times 3^2 \times (1/384) = 1/4$$

and the fundamental frequency of torsional oscillations

$$1175/\pi$$

vibrations per second.

The other values of $\sqrt{(2-\lambda_r)}$ in Table I. also have close approximations, as shown in the last column of the table.

12. Consider next the case of a four-throw crankshaft with the airscrew driven direct. Fig. 14 is a diagrammatic representation of such a system.

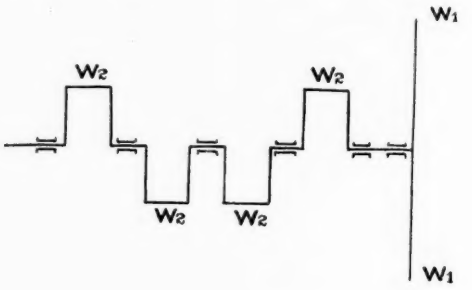


FIG. 14.

Let θ_1, θ_2, θ_3, θ_4, θ_5, be the angular displacements of the normal lines of the airscrew blades and cranks respectively (numbering from the airscrew end).

Let c_1, p_1, refer to the airscrew blades and c_2, p_2, to the cranks.

All the journals are supposed equal; c_{12} refers to the portion of the shaft between the airscrew and the nearest throw and c_{22} refers to any of the journals.

We have for the airscrew (supposed two-bladed)

$$(D^2/c_1 + 1/p_1)[c_1\phi_1 \text{ or } c_1\phi_2] = -D^2\theta_1 \quad . \quad . \quad . \quad (1)$$

From which we obtain

$$\delta_1 c_1 (\phi_1 + \phi_2) = -D^2\theta_1 \quad . \quad . \quad . \quad . \quad (2)$$

where

$$\delta_1 \equiv D^2/2c_1 + 1/2p_1$$

For the cranks

$$\delta_2 c_2 \phi_5 = -D^2\theta_2 \quad . \quad . \quad . \quad . \quad . \quad (3)$$
$$\delta_2 c_2 \phi_6 = -D^2\theta_3 \quad . \quad . \quad . \quad . \quad . \quad (4)$$
$$\delta_2 c_2 \phi_7 = -D^2\theta_4 \quad . \quad . \quad . \quad . \quad . \quad (5)$$

and

$$\delta_2 c_2 \phi_8 = -D^2\theta_5 \quad . \quad . \quad . \quad . \quad . \quad (6)$$

where

$$\delta_2 \equiv D^2/c_2 + 1/p_2$$

Also by moments

$$c_1 (\phi_1 + \phi_2) + c_{12}\theta_{12} = 0 \quad . \quad . \quad . \quad . \quad (7)$$
$$c_2\phi_5 + c_{22}\theta_{23} - c_{12}\theta_{12} = 0 \quad . \quad . \quad . \quad . \quad (8)$$
$$c_2\phi_6 + c_{22}(\theta_{34} - \theta_{23}) = 0 \quad . \quad . \quad . \quad . \quad (9)$$
$$c_2\phi_7 + c_{22}(\theta_{45} - \theta_{34}) = 0 \quad . \quad . \quad . \quad . \quad (10)$$

and

$$c_2\phi_8 - c_{22}\theta_{45} = 0 \quad . \quad . \quad . \quad . \quad . \quad (11)$$

By the elimination of the ϕ's we find

$$(D^2/c_{12} + \delta_1 + \delta_2)c_{12}\theta_{12} - \delta_2 c_{22}\theta_{23} = 0 \quad . \quad . \quad . \quad (12)$$
$$-\delta_2 c_{12}\theta_{12} + \delta_3 c_{22}\theta_{23} - \delta_2 c_{22}\theta_{34} = 0 \quad . \quad . \quad . \quad (13)$$
$$-\delta_2 c_{22}\theta_{23} + \delta_3 c_{22}\theta_{34} - \delta_2 c_{22}\theta_{45} = 0 \quad . \quad . \quad . \quad (14)$$
$$-\delta_2 c_{22}\theta_{34} + \delta_2 c_{22}\theta_{45} = 0 \quad . \quad . \quad . \quad . \quad . \quad (15)$$

where

$$\delta_3 \equiv D^2/c_{22} + 2\delta_2$$

The equation for k^2 is found to be

$$[(1/c_{12} + 1/c_2 + 1/2c_1)\,k^2 - (1/2p_1 + 1/p_2)]\,\Delta_4 - (k^2/c_2 - 1/p_2)\,\Delta_3 = 0 \qquad (16)$$

where the Δ's are as previously defined, *i.e.*,

$$\Delta_3 = K^4 - 1$$

and

$$\Delta_4 = K^6 - 2K^2$$

where

$$K^2 = [(1/c_{22} + 2/c_2)\,k^2 - 2/p_2]/(k^2/c_2 - 1/p_2)$$

Equation (16) can be written

$$[(1/c_{12} + 1/c_2 + 1/2c_1)\,k^2 - (1/2p_1 + 1/p_2)]\,A_4 - (k^2/c_2 - 1/p_2)^2\,B_4 = 0$$

where

$$A_4 = [(1/c_{22} + 1/c_2)\,k^2 - 2/p_2]^3 - 2\,(k^2/c_2 - 1/p_2)^2\,[(1/c_{22} + 1/c_2)\,k^2 - 2/p_2] \qquad (18)$$

and

$$B_4 = [(1/c_{22} + 1/c_2)\,k^2 - 2/p_2]^2 - (k^2/c_2 - 1/p_2)^2 \qquad (19)$$

There is also the frequency of the blades

$$(1/2\pi)\sqrt{(c_1/p_1)} \qquad (20)$$

and the fore and aft frequencies

$$(1/2\pi)\sqrt{(\Omega^2 + c'_1/p'_1)} \text{ and } (1/2\pi)\sqrt{(\Omega^2 + c'_2/p'_2)} \qquad (21)$$

of the blades and cranks respectively.

In the case of a four-bladed airscrew we write $2c_1$ for c_1 and $2p_1$ for p_1 in (17).

Example

Let the moment of inertia of the airscrew be very large compared with that of the estimated crank pin loads and let the flexibility of the airscrew and cranks be neglected.

The equation for k^2 will then be

$$(k^2/c_{12} - 1/p_2)\,[(k^2/c_{22} - 2/p_2)^3 - (2/p_2^2)\,(k^2/c_{22} - 2/p_2)] - (1/p_2^2)\,[(k^2/c_{22} - 2/p_2)^2 - 1/p_2^2] = 0 \qquad (1)$$

Let the dimensions of the journals be $1\frac{7}{8}'' \times 1''$ (dia.) and those of the shaft between the airscrew and the first throw $6'' \times 2''$ (dia.).

Then

$$c_{22} = (12 \times 10^6 \times \pi \times 1/16)/(2 \times 15/8) = (\pi/5) \times 10^6$$

and

$$c_{12} = (12 \times 10^6 \times \pi \times 1)/(2 \times 6)$$

i.e., $c_{12} = 5c_{22}$

Equation (1) can now be written

$$(1/5)\quad (\lambda + 3)\,\lambda\,(\lambda^2 - 2) - (\lambda^2 - 1) = 0 \qquad (2)$$

where

$$\lambda = -k^2/(c_{22}/p_2) + 2$$

The greatest positive root of (2) in λ is $\lambda = 1.822$ approximately.

Thus the fundamental frequency of the system is given by

$$k/2\pi = \sqrt{(2 - 1.822)} \times (1/2\pi)\sqrt{(c_{22}/p_2)} = (.21/\pi)\sqrt{(c_{22}/p_2)}$$

Taking the estimated crank pin load as 8lbs. at 3″

$$p_2 = (8 \times 9)/384 = 3/16$$

Hence

$$k/2\pi = (.21/\pi)\sqrt{\{(\pi/5) \times (16/3) \times 10^6\}}$$

or approximately 122 double vibrations per second.

If the engine has eight cylinders torsional resonance will occur when

$$N = (120 \times 122)/8 \text{ r.p.m.}$$

i.e., at 1,850 r.p.m. approximately.

With a four-cylinder engine with the same crankshaft dimensions and crank pin loads torsional resonance occurs at 3,700 r.p.m. approximately.

13. For a six-throw crankshaft in an ungeared engine we find the equation for k^2 is

$$[(1/c_{12} + 1/c_2 + 1/2c_1)\,k^2 - (1/2p_1 + 1/p_2)]\,\Delta_6 - (k^2/c_2 - 1/p_2)\,\Delta_5 = 0 \quad (1)$$

or

$$[(1/c_{12} + 1/c_2 + 1/2c_1)\,k^2 - (1/2p_1 + 1/p_2)]\,A_6 - (k^2/c_2 - 1/p_2)^2\,B_6 = 0 \quad (2)$$

where

$$A_6 = [(1/c_{22} + 1/c_2)\,k^2 - 2/p_2]^3 - 4\,(k^2/c_2 - 1/p_2)^2\,[(1/c_{22} + 1/c_2)\,k^2 - 2/p_2]^3 + 3\,(k^2/c_2 - 1/p_2)^4\,[(1/c_{22} + 1/c_2)\,k^2 - 2/p_2] \quad . \quad . \quad (3)$$

and

$$B_6 = [(1/c_{22} + 1/c_2)\,k^2 - 2/p_2]^4 - 3\,(k^2/c_2 - 1/p_2)^2\,[(1/c_{22} + 1/c_2)\,k^2 - 2/p_2]^2 + (k^2/c_2 - 1/p_2)^4 \quad . \quad (4)$$

For a four-bladed airscrew we write $2c_1$ for c_1 and $2p_1$ for p_1 in (1).

In addition there will be the frequencies for the blades and the fore and aft frequencies of the cranks.

14. For an eight-throw crankshaft in an ungeared engine the equation for k^2 is

$$[(1/c_{12} + 1/c_2 + 1/2c_1)\,k^2 - (1/2p_1 + 1/p_2)]\,\Delta_8 - (k^2/c_2 - 1/p_2)\,\Delta_7 = 0 \quad (1)$$

or

$$[(1/c_{12} + 1/c_2 + 1/2c_1)\,k^2 - (1/2p_1 + 1/p_2)]\,A_8 - (k^2/c_2 - 1/p_2)^2\,B_8 = 0 \quad (2)$$

where

$$A_7 = [(1/c_{22} + 1/c_2)\,k^2 - 2/p_2]^7 - 6\,(k^2/c_2 - 1/p_2)^2\,[(1/c_{22} + 1/c_2)\,k^2 - 2/p_2]^5 + 10\,(k^2/c_2 - 1/p_2)^4\,[(1/c_{22} + 1/c_2)\,k^2 - 2/p_2]^3 - 4\,(k^2/c_2 - 1/p_2)^6\,[(1/c_{22} + 1/c_2)\,k^2 - 2/p_2]$$

and

$$B_8 = [(1/c_{22} + 1/c_2)\,k^2 - 2/p_2]^6 - 5\,(k^2/c_2 - 1/p_2)^2\,[(1/c_{22} + 1/c_2)\,k^2 - 2/p_2]^4 + 6\,(k^2/c_2 - 1/p_2)^4\,[(1/c_{22} + 1/c_2)\,k^2 - 2/p_2]^2 - (k^2/c_2 - 1/p_2)^6$$

For a four-bladed airscrew we write $2c_1$ for c_1 and $2p_1$ for p_1 in (1).

In addition there will be the frequencies for the blades and the fore and aft frequencies of the cranks.

15. Generally for an ungeared engine with an n throw crankshaft the equation for k^2 will be

$$[(1/c_{12} + 1/c_2 + 1/2c_1)\,k^2 - (1/2p_1 + 1/p_2)]\,\Delta_n - (k^2/c_2 - 1/p_2)\,\Delta_{n-1} = 0 \quad (1)$$

where

$$\Delta_n = 0 \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad (2)$$

is the equation for the crankshaft system alone.

16. In the case of the airscrew blades being rigid (if this is possible) or the airscrew being replaced by a flywheel, we put

$$1/2c_1 = 0 \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad (1)$$

in the general equation in § 15 and for $2p_1$ we write p_1 where gp_1 is the moment of inertia of the flywheel about the axis of the crankshaft.

If it is desired to neglect the flexibility of the cranks we put

$$1/c_2 = 0 \quad . \quad . \quad . \quad . \quad . \quad . \quad (2)$$

in both (1) and (2) of §15.

Under these conditions we find

$$K^2 = -k^2/(c_{22}/p_2) + 2 \quad . \quad . \quad . \quad . \quad (3)$$

If in addition p_1 is extremely large two interesting cases arise.

CASE I.—All the journals equal. In this case the general equation, for an n throw crankshaft ungeared becomes

$$[-k^2/(c_{22}/p_2) + 1]\ \Delta_n - \Delta_{n-1} = 0 \quad . \quad . \quad . \quad (4)$$

or

$$K^2\Delta_n - \Delta_{n-1} - \Delta_n = 0 \quad . \quad . \quad . \quad . \quad (5)$$

or

$$\Delta_{n+1} - \Delta_n = 0 \quad . \quad . \quad . \quad . \quad . \quad (6)$$

The roots of this equation have already been discussed in §10.

CASE II.—The journal at the flywheel end, twice as stiff as the other journals. The general equation now becomes

$$[-k^2/(c_{22}/p_2) + 2]\ \Delta_n - 2\Delta_{n-1} = 0 \quad . \quad . \quad . \quad (7)$$

or

$$\Delta_{n+1} - \Delta_{n-1} = 0 \quad . \quad . \quad . \quad . \quad . \quad (8)$$

The roots of this equation have also been discussed in §10.

As an example consider the 300 h.p. Maybach aero engine. This is a six-cylinder vertical engine with a direct drive.

All the journals are equal.

The approximate dimensions of the journals are:—Outer dia. 2.6ins., inner dia. 1.4ins., and length 2.64ins.

Thus

$$c_{12} = c_{22} = \{ 12 \times 10^6 \times \pi (2.6^4 - 1.4^4) \} \ (32 \times 2.64)$$
$$= 18.84 \times 10^6 \text{ approx.}$$

The moment of inertia of a throw complete with rotating part of connecting rod is approximately 96lb. ins.^{2}.

Thus

$$p_2 = 96/384 \text{ or } \tfrac{1}{4}$$

Hence

$$\sqrt{(c_{22}/p_2)} = 8{,}700 \text{ approx.}$$

For this case we have

$$\Delta_7 - \Delta_6 = 0$$

and by §10 the lowest value for k will be the lowest value for k in

$$\Delta_{13} = 0$$

Thus approximately the fundamental frequency of the system is

$$(3/13) \times (1/2\pi)\sqrt{(c_{22}/p_2)}$$
$$\text{or } 320.$$

Torsional resonance will occur when

$$6N/120 = 320$$
$$\text{or } N = 6{,}400 \text{ r.p.m.}$$

As the engine was designed to run up to 1,500 r.p.m. there was no danger of the crankshaft failing through torsional resonance.

17. In dealing with geared engines, the treatment will depend to a great extent on the type of gearing employed. In the case of the airscrew being

geared to the crankshaft, as in Fig. 14, the treatment is comparatively simple. We will return to that case, taking into consideration the airscrew system.

Let θ_1 be the angle turned through by the crankshaft gear wheel and θ_2 that by the first crank; and let ψ_2 be the angle turned through by the normal line of the airscrew blades and ψ_1 that turned through by the airscrew shaft gear wheel.

If c_2 and p_2 refer to the cranks, we have for the crank adjacent to the gear wheel

$$(D^2/c_2 + 1/p_2)\, c_2\phi_5 = -D^2\theta_2 \quad . \quad . \quad . \quad . \quad (1)$$

and if c_1 and p_1 refer to the airscrew blades

$$(D^2/c_1 + 1/p_1)\, [c_1\phi_1 \text{ or } c_1\phi_2] = -D^2\psi_2 \quad . \quad . \quad . \quad (2)$$

Let c_{12} refer to the shaft between the first crank and the crankshaft gear wheel and c'_{12} to the airscrew shaft between the airscrew and the gear wheel.

If F is the frictional force between the gear wheels and R_1 is the radius of the crankshaft gear wheel and r_1 that of the airscrew shaft gear wheel, then

$$Fr_1 + c'_{12}\,(\psi_2 - \psi_1) = 0 \quad . \quad . \quad . \quad . \quad (3)$$

and

$$FR_1 - c_{12}\,(\theta_2 - \theta_1) = 0 \quad . \quad . \quad . \quad . \quad (4)$$

$$\therefore\ (c'_{12}/r_1)\,(\psi_2 - \psi_1) + (c_{12}/R_1)\,(\theta_2 - \theta_1) = 0 \quad . \quad . \quad (5)$$

By moments for the airscrew

$$c_1\,(\phi_1 + \phi_2) - c'_{12}\,(\psi_2 - \psi_1) = 0 \quad . \quad . \quad . \quad . \quad (6)$$

Thus by (2) and (5)

$$-2D^2\psi_2 + (r_1/R_1)\,(D^2/c_1 + 1/p_1)\, c_{12}\theta_{12} = 0 \quad . \quad . \quad (7)$$

But

$$\psi_2 = \psi_1 - (c_{12}/c'_{12})\,(r_1/R_1)\,\theta_{12} \quad . \quad . \quad . \quad . \quad (8)$$

and

$$R_1\theta_1 = r_1\psi_1 \quad . \quad . \quad . \quad . \quad . \quad . \quad (9)$$

Hence (7) becomes

$$-D^2\theta_1 + [(r_1^2/R_1^2)\,(1/2c_1 + 1/c'_{12})\, D^2 + r_1^2/2R_1^2p_1]\, c_{12}\theta_{12} = 0 \quad . \quad . \quad (10)$$

By moments for the first crank

$$c_2\phi_5 - c_{12}\theta_{12} + c_{22}\theta_{23} = 0 \quad . \quad . \quad . \quad . \quad (11)$$

so that by (1)

$$D^2\theta_2 + (D^2/c_2 + 1/p_2)\, c_{12}\theta_{12} - (D^2/c_2 + 1/p_2)\, c_{22}\theta_{23} = 0 \quad . \quad . \quad . \quad . \quad (12)$$

Hence by (11) and (12)

$$[\{1/c_{12} + 1/c_2 + (r_1^2/R_1^2)\,(1/2c_1 + 1/c'_{12})\}\ D^2 + r_1^2/2R_1^2p_1 + 1/p_2]\, c_{12}\theta_{12} - (D^2/c_2 + 1/p_2)\, c_{22}\theta_{23} = 0 \quad . \quad . \quad . \quad . \quad (13)$$

or

$$(D^2/c_{12} + \delta'_1 + \delta_2)\, c_{12}\theta_{12} - \delta_2 c_{22}\theta_{23} = 0 \quad . \quad . \quad . \quad (14)$$

where

$$\delta' \equiv (r_1^2/R_1^2)\,(1/2c_1 + 1/c'_{12})\, D^2 + r_1^2/2R_1^2p_1$$

and

$$\delta_2 \equiv D^2/c_2 + 1/p_2$$

as previously defined.

The equations for the remaining cranks will be as for the ungeared case in §12. Hence the equation for k^2 in the case of the four-throw crankshaft with the engine geared as in Fig. 14 (the airscrew being two-bladed) is

$$[\{1/c_{12} + 1/c_2 + (r_1^2/R_1^2)\,(1/2c_1 + 1/c'_{12})\}\ k^2 - (r_1^2/2R_1^2p_1 + 1/p_2)]\,\Delta_4 - (k^2/c_2 - 1/p_2)\,\Delta_3 = 0 \quad . \quad . \quad . \quad . \quad (15)$$

where the Δ's are as previously defined. The equation (15) can be written

$$[\{1/c_{12} + 1/c_2 + (r_1^2/R_1^2)(1/2c_1 + 1/c'_{12})\}\ k^2 - (r_1^2/2R_1^2 p_1 + 1/p_2)]A_4 - (k^2/c_2 - 1/p_2)^2 B_4 = 0 \quad . \quad . \quad . \quad . \quad (16)$$

where A_4 and B_4 are as in the ungeared case in §12.

Similarly for the case of a six-throw crankshaft, geared as in Fig. 14, the equation for k^2 is

$$[\{1/c_{12} + 1/c_2 + (r_1^2/R_1^2)(1/2c_1 + 1/c'_{12})\}\ k^2 - (r_1^2/2R_1^2 p_1 + 1/p_2)]A_6 - (k^2/c_2 - 1/p_2)^2 B_6 = 0 \quad . \quad . \quad . \quad . \quad (17)$$

where A_6 B_6 are as defined in §13; and for an eight-throw crankshaft we have A_8 and B_8 as defined in §14.

CORRESPONDENCE

To the Editor of the JOURNAL OF THE ROYAL AERONAUTICAL SOCIETY.

SIR,—The notable addition to the Society's library of a selection of rare and interesting books relating to early phases of aeronautics (as described in the article reprinted in last month's Journal from the "Times" Literary Supplement) is rightly regarded as a matter of congratulation. May I be allowed, as one greatly interested in aeronautical history, to offer a few observations on the two-fold interest of this acquisition?

Firstly, it may be claimed that the purchase of these books for the Society's library is of consequence, inasmuch as it tends to make the library—what it surely ought to become—the most important collection of historical and technical books, English and foreign, in all branches of that science which it is the aim of the Society to further by every means in its power. If it be said—as one has heard it said of collections of early works on other sciences—that the older books are seldom referred to, it may be answered that the prestige of any scientific institution receives permanent support from the possession of an important and well-arranged library of reference. The acquisition for the library of the Institution of Electrical Engineers of the remarkable collection of rare early works on electricity and magnetism, formed by the late Professor Sylvanus Thompson, may be cited as a signal example.

Secondly, the possession of a library of great and increasing historical value may well prove to be a matter of more direct practical importance, should it lead the Society to encourage a study of the historical aspects of aeronautical history. Hitherto, very little serious work of that character has been done in England—the publication by the Society of the series of "Aeronautical Classics" is hardly more than an admirable exception which goes to prove the rule. Encouragement is at the present time the more desirable in view of the marked attention which has been paid in recent years to the study of historical science and technology in all directions. Important work of this kind has been done, and is now being done, in England—examples will readily occur to all students of science—and it is therefore unnecessary to particularise. The remark holds good of the Continent, and in the United States there has been remarkable activity on similar lines of historical research, covering the more important fields of applied science and technical industries. Aeronautical science, though the youngest regarded from the point of view of practical achievement, is destined for years to come to be in the van of progress. Can it afford to stand apart from the endeavours now being made to satisfy the need for historical knowledge of a technical character by neglecting the study of its own history—a history which, though scant in important material, and interminably slow in the tale of its unfolding, is yet of undeniable interest, and should be allowed its rightful place in the annals of science and engineering?

Up to the present, however, it must be admitted that in England scholarship has almost wholly ignored, or at the least shown a marked indifference to, the historical aspects of aeronautics. It is significant that to this day Hatton Turnor's cumbrous and ill-arranged compilation, "Astra Castra" (published in 1865), is perforce still frequently referred to as the "best" and "fullest" authority. That it has been quoted by countless writers may be fully admitted, but it is difficult to believe that those who cite it so freely (though not always by name) ever studied the subject at first hand. As a matter of fact, there is very little original work in Turnor's volume—of the 518 pages, about 350 consist of tedious padding in the form of poetical and Scriptural quotations, or wholesale

reprints of earlier books and essays, made for the most part without any comment or connected narrative. Further, it contains a full measure of inaccuracies and notable omissions, and a bibliography which is hopelessly inadequate.

Moreover—not to attempt a survey of aeronautical literature—the D.N.B., though it includes such well-known pioneers of the free balloon as Vincenzo Lunardi (who rightly described himself as a child of Britain only by a species of adoption), Charles Green, George Gale (whose inclusion it would be hard to justify), and Henry Coxwell, omits such important names in the evolution of British aeronautical science as Sir George Cayley (the full extent of whose work in both branches of aeronautics—dirigible balloons and mechanical flight—has not yet been adequately noticed), John Stringfellow, F. H. Wenham and Percy S. Pilcher. Indeed, it must be said that English aeronautical books dealing with the historical aspects are, broadly speaking, unscholarly, compiled with a minimum of research, unreliable as to facts, and usually lacking in authorities, while they fail to convey any large view of the lines on which aeronautical science evolved.

On the other hand, France has been far better served by her scholars and scientists. Faujas de Saint-Fond set an example as early as 1783 in his scientific account of the first aerostatic achievements, an example followed in later times by Dupuis-Delcourt, Landelle, Tissandier (whose fine "Histoire des Ballons" is, however, marred by too frequent errors), Lecornu, and others, all of whose writings are distinctly in advance of any similar works published in England. Nor can we boast anything to compare with the sumptuously illustrated volumes—designed as pictorial histories of flight—published in Paris under the editorship of M. F.-L. Bruel (1909) and the Comte de la Vaux (1922), or with the comprehensive and accurate I.L.A. Catalogue compiled by Dr. Liebmann and Dr. Wahl in connection with the Aeronautical Exhibition at Frankfort in 1909. Italy, also, has to her credit the scholarly historical monograph "Il Volo in Italia" by Guiseppe Boffito, published in Florence two years ago, while a more modest but useful little work by E. Fuld was privately printed at Amsterdam in 1918 under the title of "Uit de eerste jaren der Luchtvaart in Nederland."

In face of these facts—capable of amplification, but of themselves surely not creditable to a country which has done great things in the air, or to an Empire which must ever be as deeply concerned in aerial as it is in maritime navigation—may I respectfully urge the Council of the Society to encourage, if they cannot actively promote, researches into the history of aeronautical science, to which the admirable acquisition herein discussed may be deemed to serve as a fitting prelude. Would it, for instance be possible to afford a brief space in the pages of the Journal for limited excursions into the technology of aeronautics? Or if the regrettably high cost of printing precludes even occasional essays in that direction—and it will be generally allowed that the invaluable papers recording the most recent results of aeronautical research, as printed in the Journal from month to month, constitute a very important part of the Society's work—might it not exert some influence by inviting the co-operation of the Newcomen Society (whose excellent work on the history of engineering and technology is deserving of all possible support) in discussions on subjects of aeronautical interest? As a lay member of both Societies I throw out these ideas unofficially and with all diffidence, but not without hope of some result.

And may it not be added there is reason to believe that endeavours on the lines above mentioned would tend to have the incidental but most desirable result of stimulating public interest in aeronautical matters at large? One's limited experience in lecturing on historical aeronautics confirms the impression that in this, as in other matters, the regrettable lack of interest is due, partly at least, to lack of knowledge.—Yours faithfully,

J. E. HODGSON.

Bromley.
March 12th, 1923.

REVIEWS

Aerodynamik

R. Fuchs and L. Hopf (R. C. Schmidt & Co., Berlin).

In this volume the authors aim at a complete account of the aerodynamics of the aeroplane which may serve as a basis for any further work either on practical or on theoretical lines, and they have undoubtedly produced a most interesting and valuable book on this subject. The book is divided into two main parts, the first dealing with the physical theory of air forces and the second with the motion and stability of an aeroplane.

After a short introductory chapter on the methods of classical hydrodynamics, a very full account is given of the problem of the two dimensional aerofoil, including the well-known series of Joukowski aerofoils and the extended types which have been discussed more recently by Karman and Trefftz. The next two chapters deal with Prandtl's theory of finite aerofoils on lines characteristic of Dr. Fuchs himself. The fifth chapter is entitled the "Theory of Drag," and contains an excellent and full account of the theoretical development of this subject. The theory of drag forces is the weakest part of aerodynamic theory, but the various attempts to obtain an analytical solution of the problem are reproduced. Helmholtz' surfaces of discontinuity, Karman's vortex rows and Prandtl's viscous boundary layer all throw some light on this obscure problem and indicate the lines of further progress even if they do not as yet provide a method of calculating the drag of a body.

The theoretical work is followed by a long chapter containing typical experimental results relating to scale effect, the forces on aerofoil structures, tail planes and other aeroplane parts, and concluding with a detailed analysis of the drag of two aeroplanes. The first part of the book concludes with a short and rather unsatisfactory outline of airscrew theory.

The second part of the book, due mainly to Dr. Hopf, discusses the motion and stability of an aeroplane. The opening chapter deals with performance, atmospheric conditions, methods of test and of analysis, and concludes with an analysis of the component weights of a large number of aeroplanes. The second chapter is concerned with the balance of the pitching moment of an aeroplane, and is a very full treatment of the problem, including a criticism of the analogy of the metacentric height of a ship which is so often used in a fallacious manner. The last two chapters discuss very fully the general longitudinal and lateral motion of an aeroplane, including the stability of straight flight, the equilibrium of circling flight, and the problems of spinning and autorotation. The treatment of these problems is quite satisfactory, and, although differing in formal presentation, is essentially identical with current English practice, to which indeed the author expresses his indebtedness in several particulars.

The whole book is well composed, and is a continual inspiration to further investigation. There are unfortunately a number of misprints which increase the difficulty of understanding the mathematical analysis, and it is regrettable that so good a book should contain the statement that the centre of pressure of a wing goes to infinity when the lift vanishes. These are, however, only minor points in an excellent book which should become one of the classical works on aerodynamics.

THE ROYAL AERONAUTICAL SOCIETY

ANNUAL GENERAL MEETING

The Fifty-eighth Annual General Meeting of the Society was held in the Society's offices, 7, Albemarle Street, London, at 5 p.m., on Tuesday, March 27th, 1923. The Chairman, Professor L. Bairstow, C.B.E., F.R.S., presided, and the following were present :—

Council.—Sir Mackenzie Chalmers, Colonel The Master of Sempill, Major R. V. Southwell.

Members.—R. M. Balston, Commander F. L. M. Boothby, C. F. Dendy Marshall, Lieut.-Colonel S. Heckstall Smith, Major D. H. Kennedy, J. L. Lake, Major A. R. Low, Lieut.-Colonel V. C. Richmond and Dr. H. C. Watts.

Commander Boothby, Mr. Dendy Marshall and Major Kennedy were appointed scrutineers of the ballot for the election of Council.

The Chairman, in presenting the Council's report and balance sheet, said that it was of great interest that the organisation of the International Air Congress, which is to be held in London this year, is to be carried on here, a special grant which will cover the estimated expenditure having been made for the purpose.

He called attention to the grant which had been made to the Society by the Trustees of the Carnegie United Kingdom Trust Fund, which had made it possible to purchase a very valuable collection of early aeronautical books, and also more modern ones, badly needed to complete the Society's collection. The Secretary had found that these rare books were in the hands of Messrs. Maggs, and the Council mentioned the matter to Lord Weir, who approached the Trustees of the Carnegie Fund, with the result that a sum of £500 was granted to the Society, most of which had been expended in the manner described, while the rest would be spent during the coming year. The Trustees had made very few conditions with regard to the grant, the most important being that the Society should agree to lend the books if applied for through the Central Library for Students.

On the whole, the position as regards Membership of the Society was rather more satisfactory than it had been during the last two or three years. Instead of the membership falling, the total for the year showed a slight increase. Subscriptions are difficult to get in, but on the whole he considered the position of the Society far better than it was last year.

The Society's custom of awarding medals for papers had been revived during the year under review, and Mr. H. R. Ricardo was awarded the Silver Medal for 1922 for his paper on "Some Possible Lines of Development in Aircraft Engines." That medal is given to the author of the best paper published in the Journal during each year, but it should be explained that the Council, who decide on the merits of the various papers, do not consider it desirable to award this prize to Members of Council, who are therefore debarred from an award under that heading.

The R.38 Memorial Research Fund had now reached a total of £1,264 19s. 11d., and the Council had decided to devote the Fund in part to the erection of a memorial tablet in the Library over the mantelshelf. This work

was in the hands of Mr. Paul Cooper, who hoped shortly to show the Council a small model of his design. Part of the income of the Fund is to be devoted to a prize of twenty-five guineas annually for the best paper dealing with some technical subject in aeronautics, preferably in connection with airships, to which there had been a good response. Although the entries had to be made before December 31st last year, twelve entries had been received (British, German, Italian and American), and it was interesting to find that they were all connected with airship problems.

The last year had seen the institution of the Society's examination for Associate Fellowship. Not many entries were made, but the important point was that the whole machinery for the examination is now ready so that the Society can deal with any Students who come forward. The fees decided upon barely covered expenses, but University practice had been followed, and it was not proposed to alter them.

Another new feature was the holding of a Technical Discussion. So far there had been two very good meetings on the "Prandtl Theory of the Flow Round an Aerofoil." Mr. Glauert wrote a paper to start the discussion, which had been put into print. At both meetings several people spoke, and now three or four papers were coming along ready for printing. When the whole discussion was published in the Journal it would give an interesting account of the various opinions held on this subject. By no means a unanimous view would appear, but the effect of the discussion had been to show just where the Prandtl theory is useful, and just where suspicion about it arises.

Other subjects had been suggested to follow this discussion, the holding of which had been very well attended. On both occasions there was considerable difficulty in finding seating accommodation in the Society's Library for the number of people present.

The title of the Journal had been changed at the end of the year, a certain amount of difficulty having arisen from the fact that the title of "The Aeronautical Journal" did not disclose that it had any connection with the Royal Aeronautical Society. The circulation had increased, owing to efforts made by the Editor and Secretary. A good many of the American Air Service stations now received it officially, and it was also supplied by the Air Ministry to Royal Air Force units. Mention of the Journal reminded him that he should like to bring to the notice of the meeting the services of Mr. J. L. Pritchard, who has acted as Editor of the Journal for some years. He was appointed in 1919 at a salary of £50, and that salary he had never accepted, or had returned it as a donation to the Society, and the Council felt very strongly that they were greatly indebted to him for getting the Journal into a good form and improving it technically. He thought most of the members present must have noticed its improved appearance and form.

The Students' Section had been very active. It had made visits to various works, and had held several meetings. It had also branched out in an entirely new line by holding its first social function—a smoking concert—and this was to be followed by another concert to which other grades of the Society's membership were invited.

With regard to the Balance Sheet, if Members would look at the bottom of this they would find the whole summarised by an excess of expenditure over income of over £278. That was an unsatisfactory state of affairs in some ways, but it did not mean that the Society was not improving its position. The best way to get rid of the deficit would be to improve the number of members. With more members the total cost of Journals went up, but the cost of salaries did not, and the Society was in a position greatly to expand its membership at very slight expense. Of course, if the aeronautical industry were in a more flourishing

condition, the Society's membership would soon feel a response. The Balance Sheet had been approved by the members of Aerial Science Limited.

The adoption of the Report was proposed by Major Low, seconded by Dr. Watts, and carried unanimously.

The scrutineers then presented the result of the ballot, and the following members were declared elected to serve on the Council for the two years ending March, 1925 :—

Professor L. Bairstow, C.B.E., F.R.S., Wing Commander T. R. Cave-Browne-Cave, C.B.E., Sir Mackenzie Chalmers, K.C.B., C.S.I., Mr. C. R. Fairey, C.B.E., Lieut.-Colonel J. T. C. Moore-Brabazon, M.C., M.P., Lieut.-Colonel M. O'Gorman, C.B., D.Sc., Mr. J. L. Pritchard, Colonel The Master of Sempill, A.F.C., Major R. V. Southwell, and Lieut.-Colonel H. T. Tizard, A.F.C.

A vote of thanks to the scrutineers was proposed by Professor Bairstow and seconded by Colonel The Master of Sempill, and the meeting closed with a vote of thanks to the Chairman proposed by Mr. R. M. Balston and seconded by Major Kennedy.